Spin Waves & Neutron Scattering

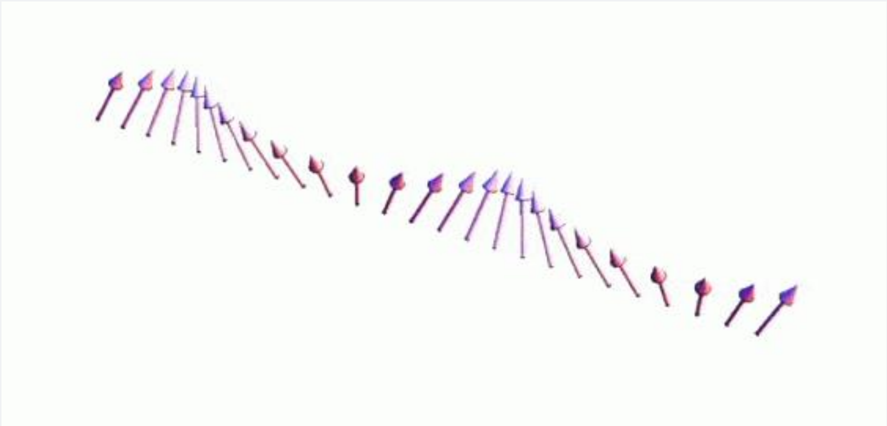
By Cory Trout

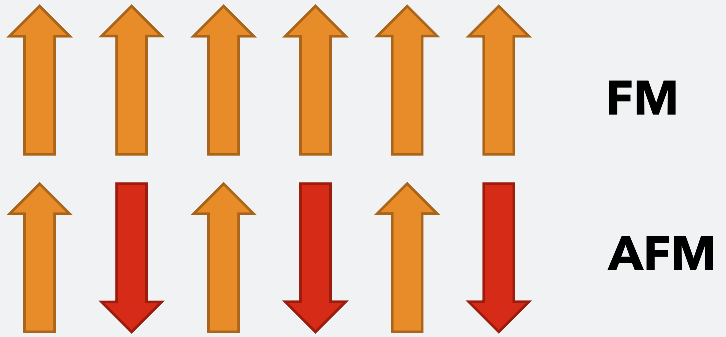
Outline

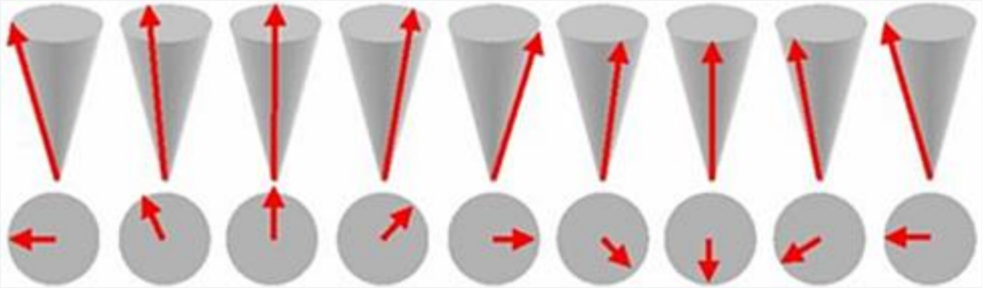
- Motivate spin waves from a theoretical perspective
  - Ferromagnets & Antiferromagnets
- How can we detect the presence of spin waves
  - Neutron Scattering
- Physical examples in Iron and Manganese Fluoride

What are Spin Waves?

- Spin waves are low energy excitations of magnetic materials which manifest themselves as a propagating disturbance in the magnetic ordering of a material.
- Spin waves occur in ordered materials such as ferromagnetic (FM) or antiferromagnets (AFM)
- The 1D case offers some insight into the behavior of low energy excitations of magnetic materials



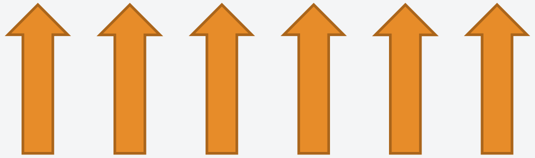




Motivation for finding Spin Waves

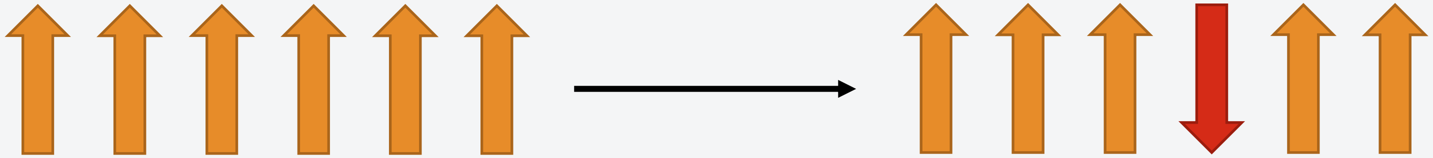
Let's examine the simplest case which is a 1D ferromagnetic chain in which the ground state has all of the spins aligned

The energy of the ferromagnetic ground state can be easily calculated with the Heisenberg Hamiltonian assuming only nearest neighbor interactions



$$H = -J\sum_{\langle ij\rangle} S_i \cdot S_j = -2J\sum_i S_i \cdot S_{i+1} = -2JNS^2$$

We are interested in finding the lowest energy excitation of this simple case. A good first guess might be the energy a single spin flipped



$$H = -2J\sum_i S_i \cdot S_{i+1} = -2J(N-2)S^2 + 2(2JS_i \cdot S_{i+1} + 2JS_{i-1} \cdot S_i) = E_o + 8JS^2$$

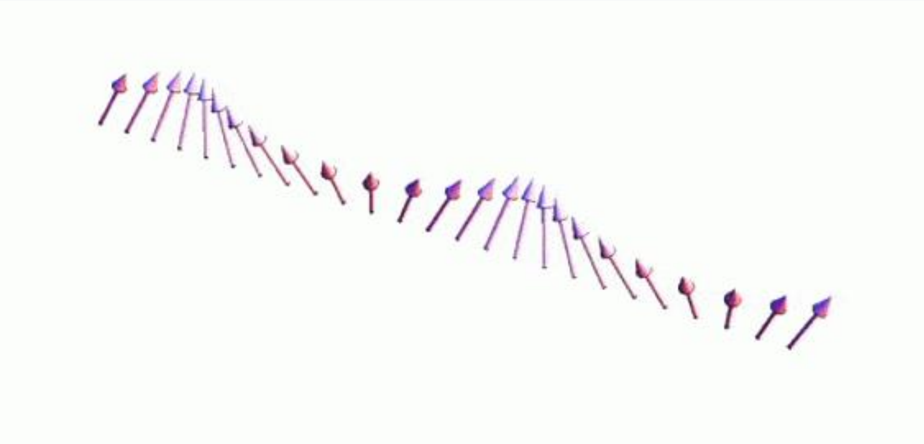
Is this actually the lowest energy excitation? It turns out the answer is no!

Construction of a spin wave state

How can we construct a lower energy state?

Semi-classically, we can consider each spin precessing about their ground state orientation

$$\hbar \frac{d\vec{S}_j}{dt} = \vec{\mu} \times \vec{B}_{eff}$$



Quantum Mechanically, we can borrow the semi-classical description and consider a superposition of states that "share" the spin flip across the chain

Let's try a Fourier expansion of states

$$|q\rangle = \frac{1}{\sqrt{N}} \sum_j e^{iqR_j} |j\rangle$$

Where $|j\rangle$ is the state of the entire chain

Example:

$$|j\rangle = | \uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow \cdots \rangle$$

Dispersion of Spin Wave

Although it is beyond the scope of the presentation to fully derive the dispersion of a spin wave, we can outline the procedure

In this new representation of the states of the 1D chain ($|j\rangle = |\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\cdots\rangle$), it is convenient to work with the ladder operators

Ladder Operators

$$S^+ = S^x + iS^y$$

$$S^- = S^x - iS^y$$

$$S^2 = \frac{1}{2}(S^+S^- + S^-S^+) + S_z^2$$

$$H = -2J\sum_i S_i \cdot S_{i+1} \longrightarrow H = -2J\sum_i \left[S_i^z S_{i+1}^z + \frac{1}{2}\left(S_i^+ S_{i+1}^- S_i^- S_{i+1}^+\right)\right]$$

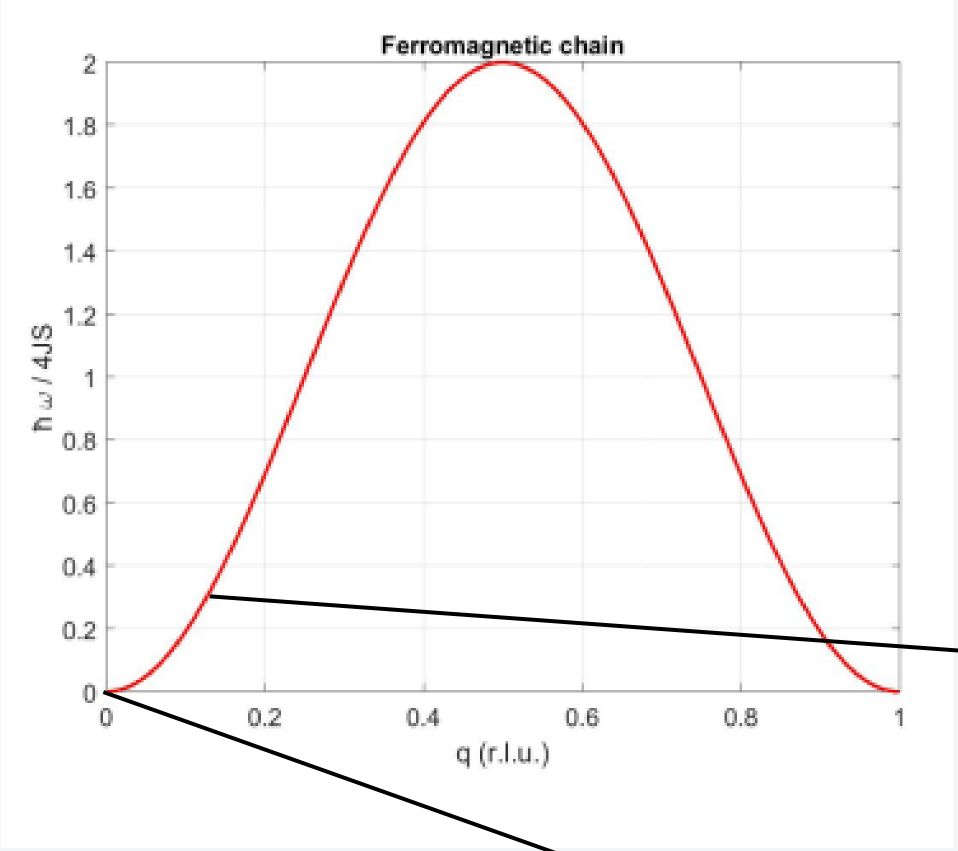
Now we can find the dispersion relation by

$$H|q\rangle = E(q)\frac{1}{\sqrt{N}}\sum_j e^{iqR_j}\,|j\rangle$$

The ladder operations makes this an easy calculation, and we find

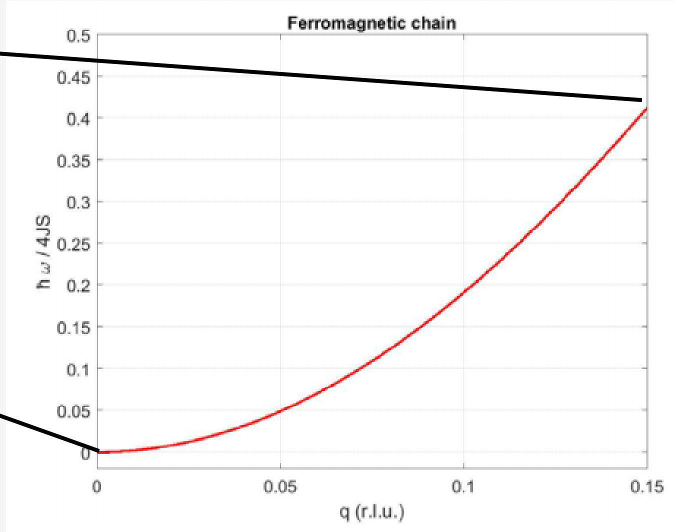
$$E(q) = -2NS^2J + 4JS(1 - \cos(qa))$$

Dispersion of Spin Wave in Ferromagnetic Chain



$$E(q) = -2NS^2J + 4JS(1 - \cos(qa))$$

We see that the excited state is just the ground state plus a small perturbation

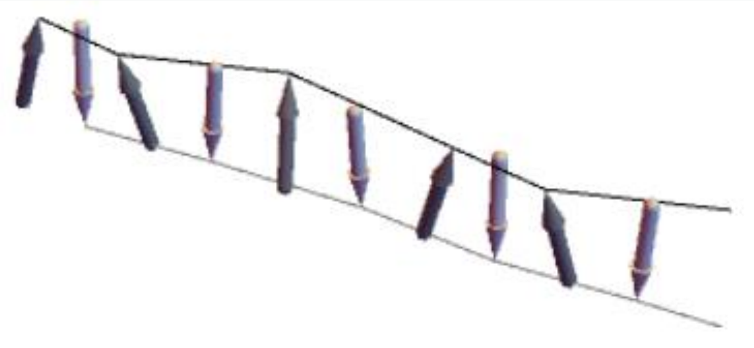


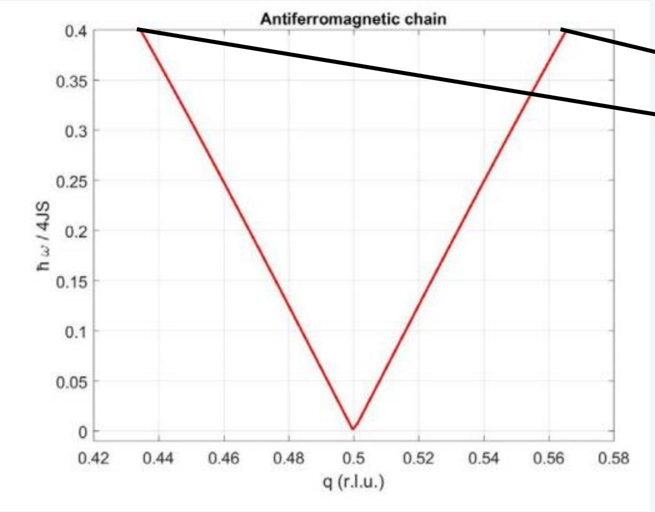
Quadratic dependance at low wavenumber

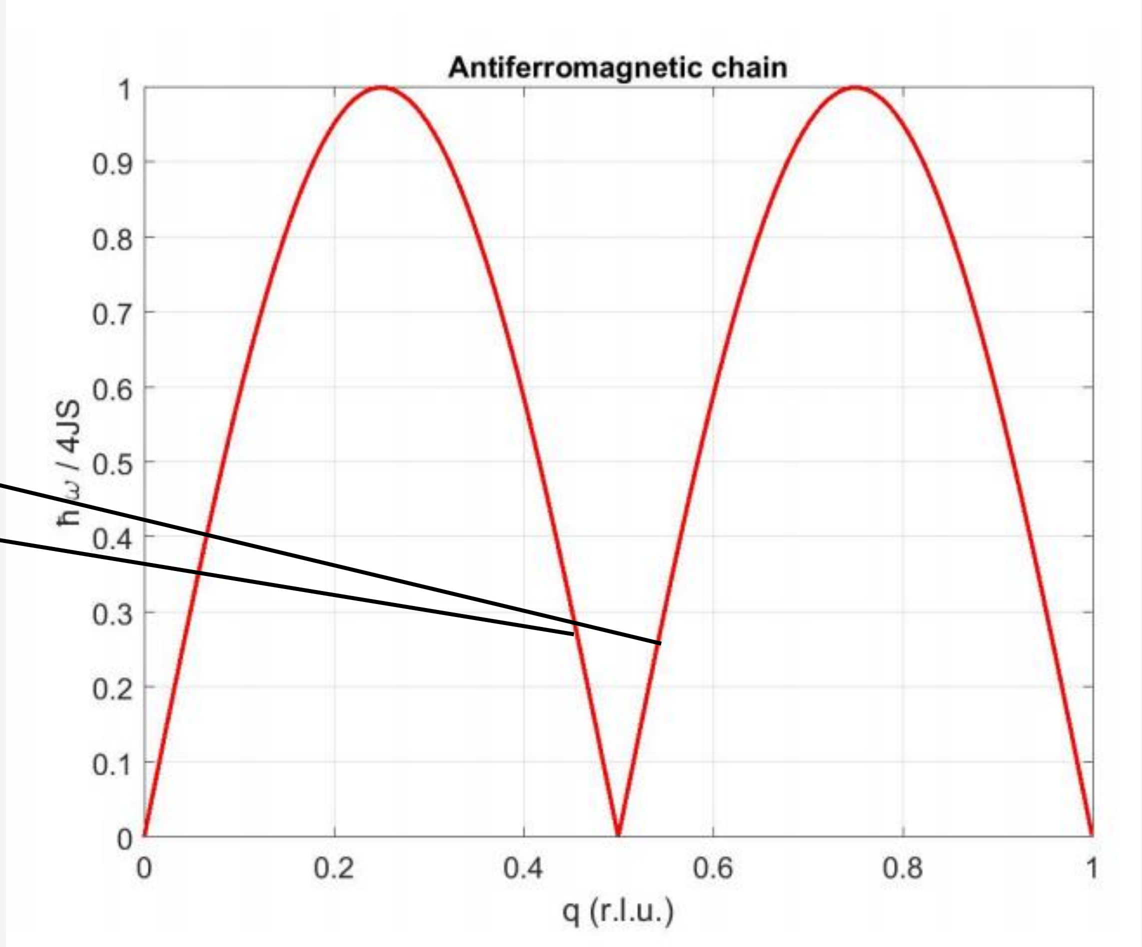
Spin Waves in 1D Antiferromagnetic Chain

Spin waves can also be present in antiferromagnetic materials as well, but in general this is more difficult to solve. The semi-classical approach is most common and easiest to compute. Here we will only discuss the results.

$$E(q) = 4JS|\sin(qa)|$$



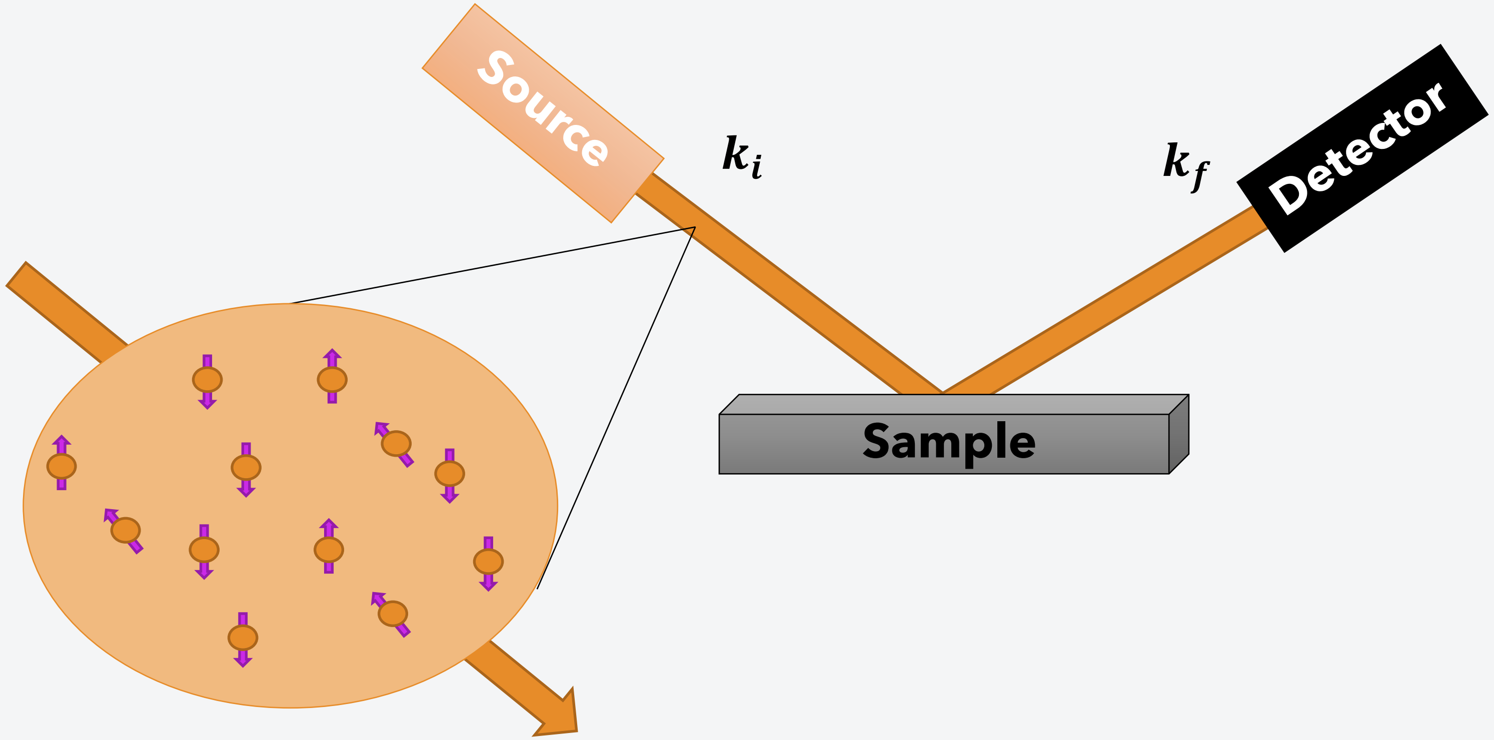




We see the dispersion is approximately linear for small q

Neutron Scattering

One of the most prominent ways to characterize magnetic materials is neutron scattering. A huge contributing factor is the fact that neutrons have spin! This allows neutrons to investigate the magnetic structure of materials as well as magnetic excitations.



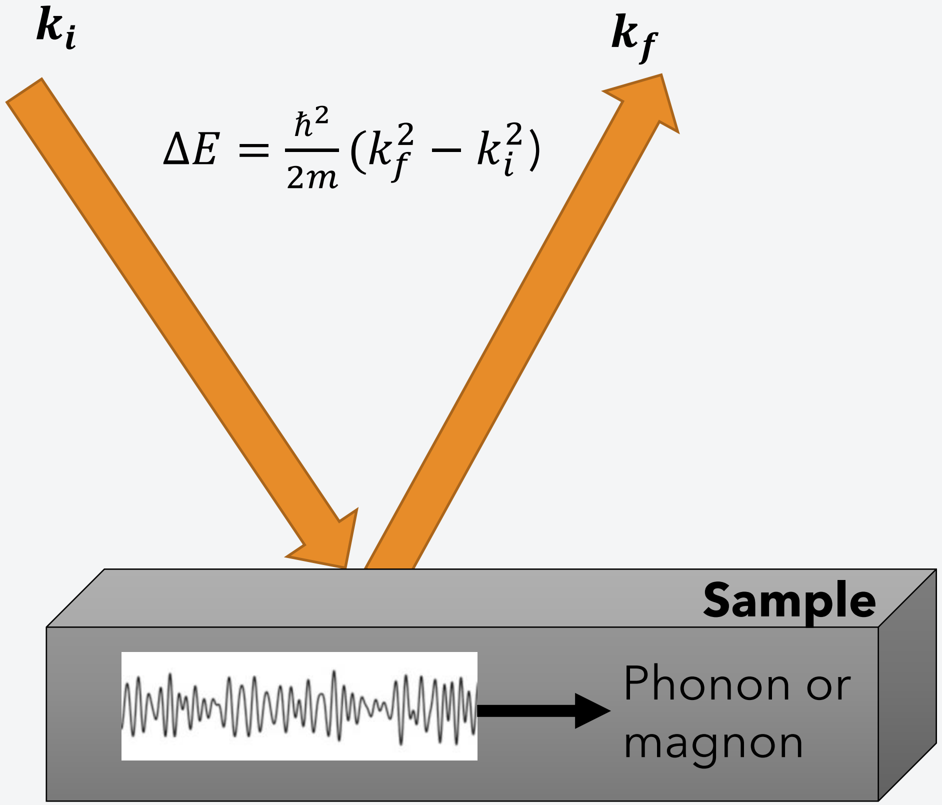
Elastic

- $k_i = k_f$
- No Energy transfer
- Measure Bragg Peaks
- Determine structure
    - Magnetic structure

Inelastic

- $k_i \neq k_f$
- Energy transfer ($\Delta E = \frac{\hbar^2}{2m}(k_f^2 - k_i^2)$)
- Ability to measure excitations
- Investigate phonons and magnons

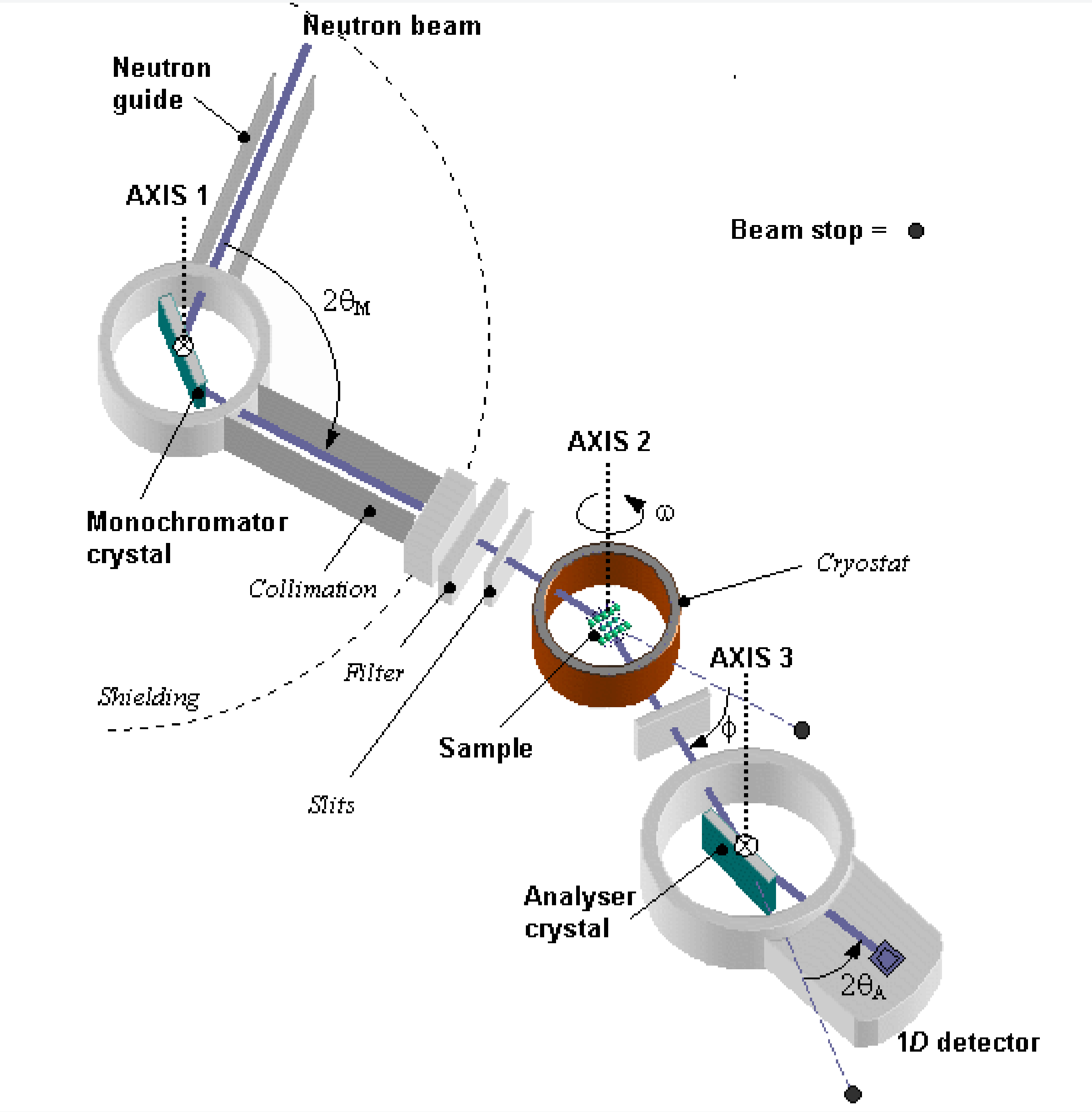
Inelastic Neutron Scattering



The energy lost in the neutron beam is deposited into the sample. It is this energy that can excite spin waves.

By counting the intensity of neutrons at wavenumbers shifted from the initial neutrons, the spin waves can be measured

Triple Axis Spectrometer



- The first rotation axis contains a crystal is monochromate the neutron beam
- The second axis contains the sample
- The third axis contains an analyzer crystal to select the outgoing energy of the scattered neutrons

Spin Waves

E. J. KONDORSKY AND J. R. SCHRIEFFER, *Chairmen*

Spin Waves in 3*d* Metals*

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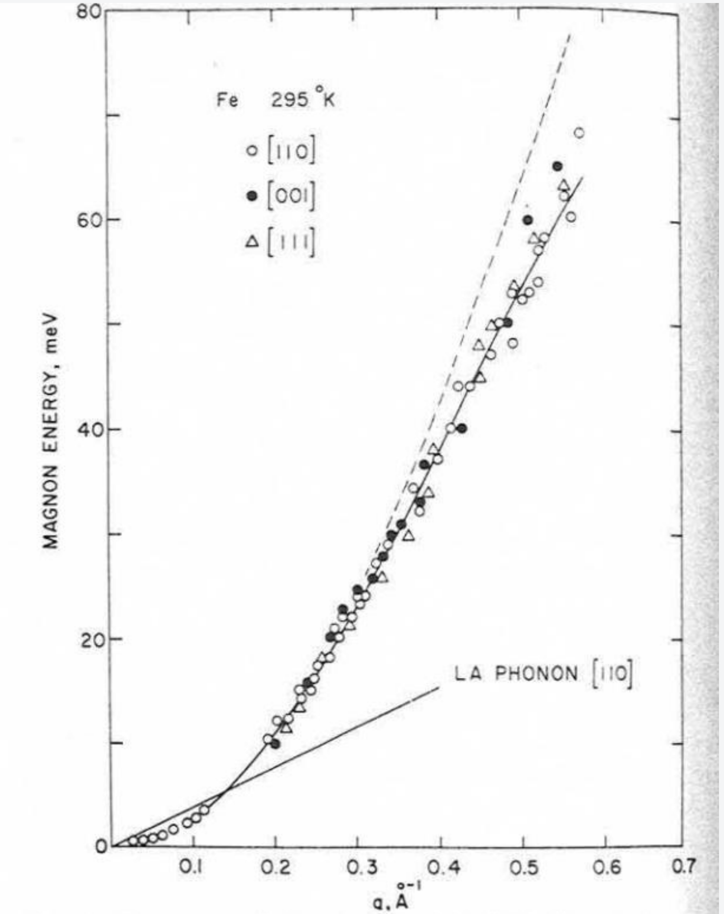


FIG. 4. Dispersion relation for Fe at 295°K. The broken line corresponds to the Heisenberg model with $D=281$ meV Å².

Dispersion relation of ferromagnetic iron is shown to have quadradic dependance at low wavenumbers

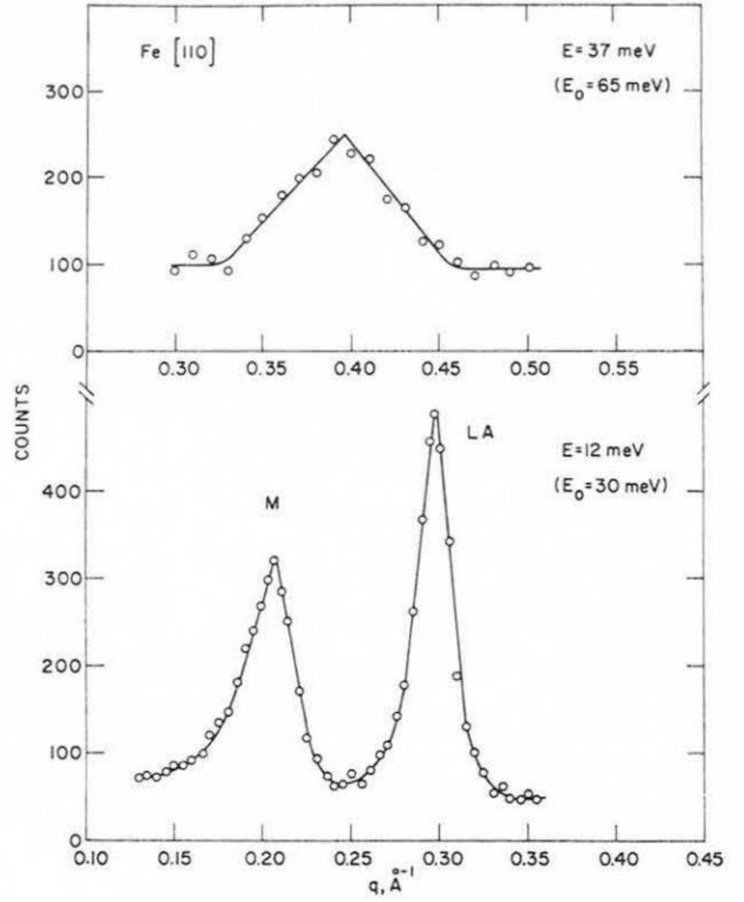
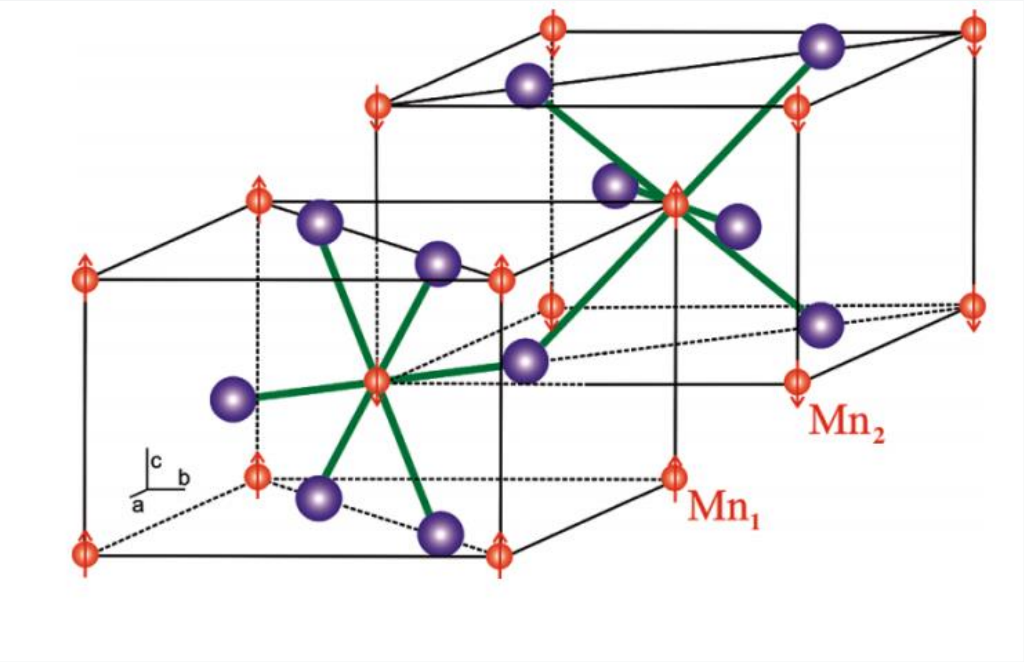


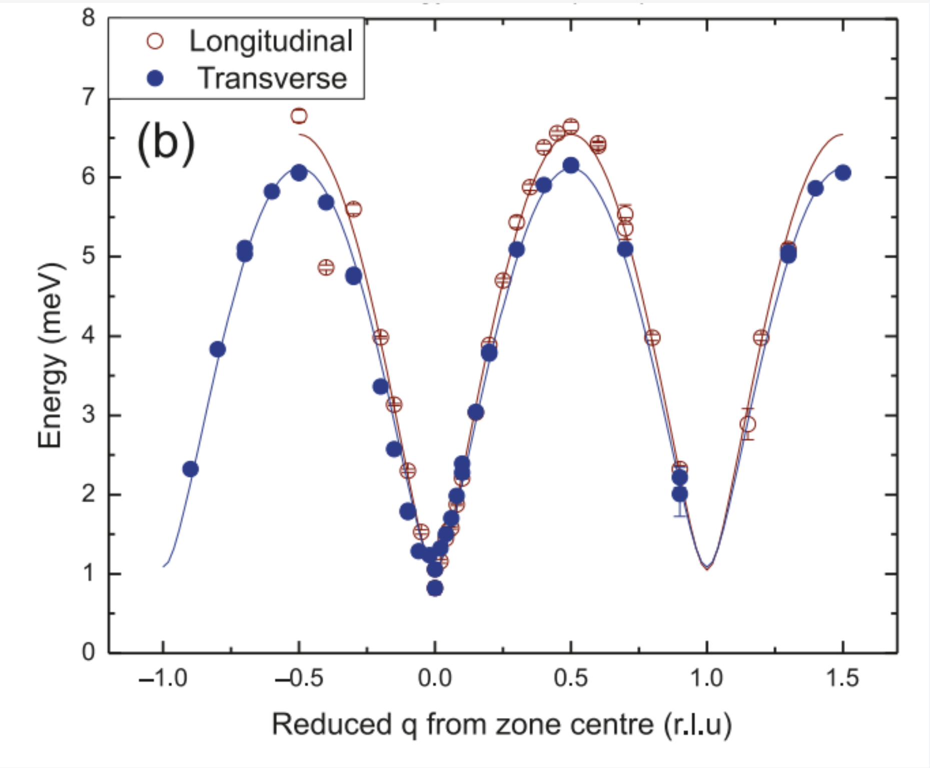
FIG. 2. Contant E scans of spin waves (M) and longitudinal acoustic (LA) phonons in Fe at 295°K with fixed incoming energy E_0.

Inelastic neutron scattering using triple axis spectrometry detect spin waves and phonons in iron

Neutron scattering study of the classical antiferromagnet MnF_2: a perfect hands-on neutron scattering teaching course[1]

Z. Yamani, Z. Tun, and D.H. Ryan





THANK YOU FOR LISTENING
Questions?