Aharonov-Bohm Effect

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Outline

- Experimental Setup
- Aharonov-Bohm Effect
- Berry Phases

Goal

- The potential fields in quantum mechanics are physically relevant

What is it?

- For dynamics we consider fields *($\vec{E}$* or *$\vec{B}$)* real

$$m\vec{a} = q(\vec{E} + \vec{v}\times\vec{B})$$

- Potential fields are auxiliary *(ϕ or $\vec{A}$)*

$$\vec{E} = -\vec{\nabla}\phi, \vec{B} = \vec{\nabla}\times\vec{A}$$

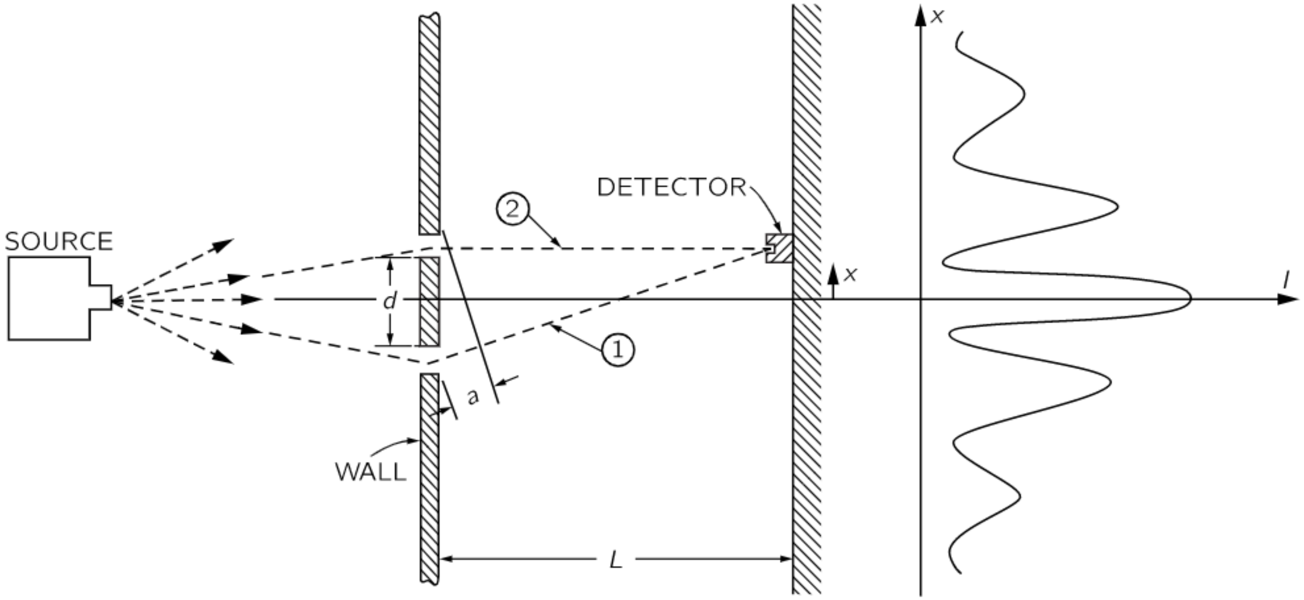
Aharonov and Bohm showed in quantum mechanics both are physical

- Without a field $\vec{B}$*,* non-zero $\vec{A}$ affects

https://www.feynmanlectures.caltech.edu/II_15.html#Ch15-S5difference

Wave Exposition

Consider a double-slit with light being emitted.
Two plane waves:



$$\psi_1(r,t) = A_1 e^{i(\phi_1(r)-\omega t)}$$

$$\psi_2(r,t) = A_2 e^{i(\phi_2(r)-\omega t)}$$

Superposition: $\psi(r) = \psi_1(r,t) + \psi_2(r,t)$
Intensity at detector is

$$I = \int \psi(r,t)\psi^*(r,t)dt \ \propto A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2)$$

Interference is given by difference in phase $\phi_1 - \phi_2$

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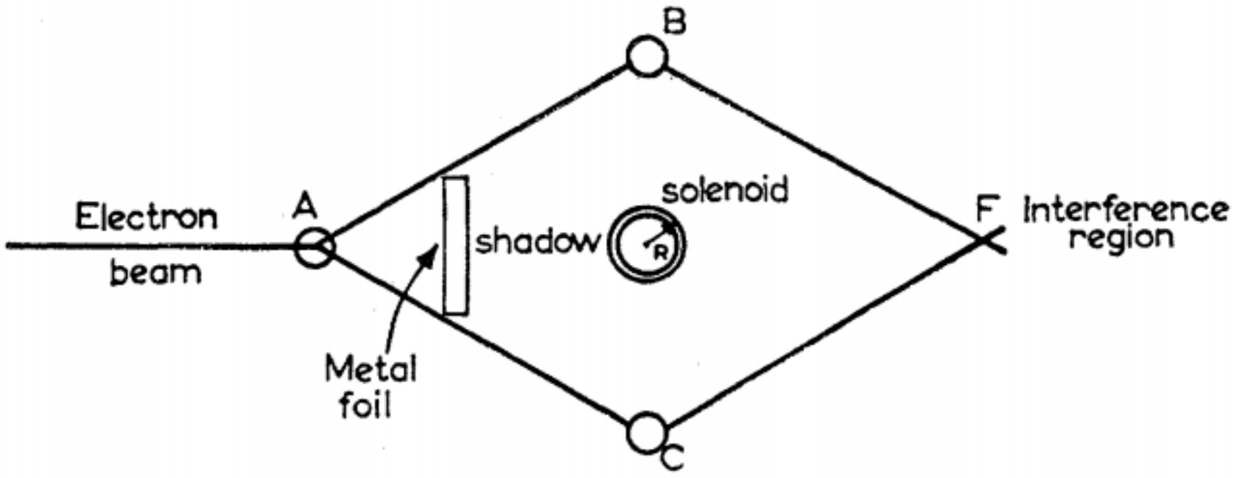
Solenoid Example

Consider single electron split at A.

With planewave solution of



$$\psi_B(r) = Ae^{i\phi_B(r)}$$

$$\psi_C(r) = Ce^{i\phi_C(r)}, \qquad \psi(r) = \psi_B(r) + \psi_C(r)$$

At the detector at F, interference given by $\phi_B - \phi_C$

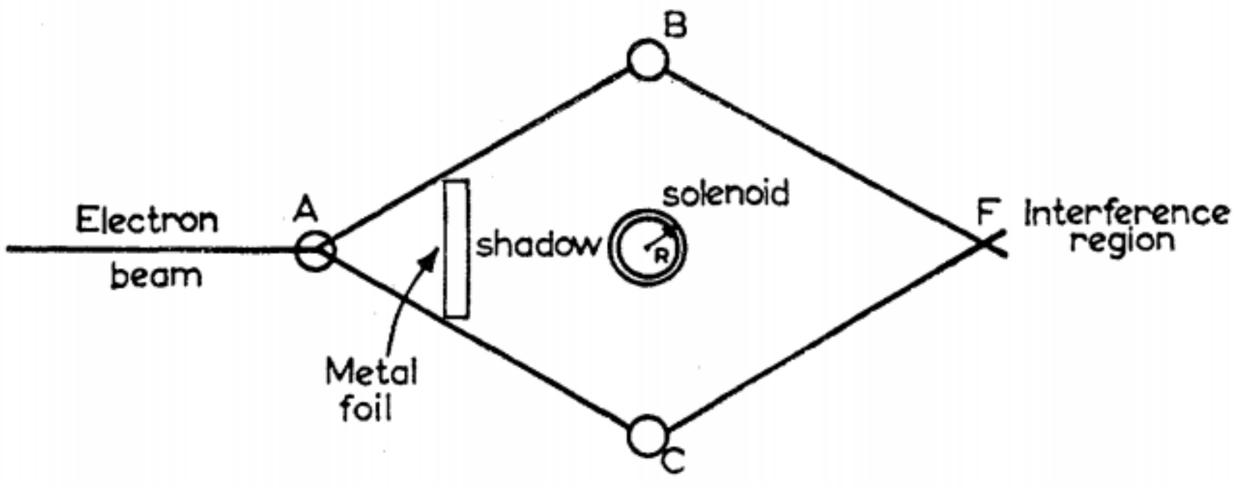
Interference depends on the vector potential from the solenoid.

Aharonov and D Bohm. *Significance of Electromagnetic Potentials in the Quantum Theory.*

Solenoid Example

Hamiltonian with magnetism:

$$\hat{H} = \frac{1}{2m}\left(-i\hbar\vec{\nabla} + e\vec{A}(\vec{r})\right)^2$$



$\vec{A}(\vec{r})$ is non-zero outside solenoid.

Then the groundstate solution is:

$$\psi(r) = e^{-\frac{ie}{\hbar}\int_{r_A}^{r} \vec{A}(\vec{r}\,')\cdot d\vec{r}\,'}\psi(r_o)$$

Identify

$$\phi = -\frac{e}{\hbar}\int_{r_A}^{r} \vec{A}(\vec{r}\,') \cdot d\vec{r}\,'$$

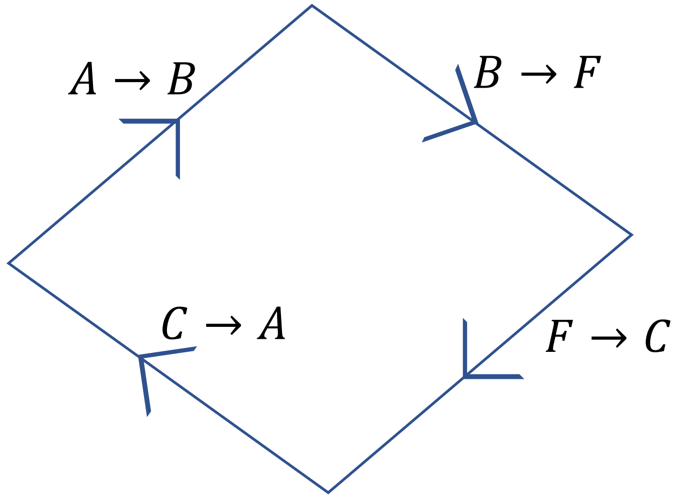
Aharonov and D Bohm. *Significance of Electromagnetic Potentials in the Quantum Theory. PRL, 1959.*

Aharonov- Bohm Effect

Interference is then

$$\phi_B - \phi_C = -\frac{e}{\hbar}\int_{r_A}^{r_F} \vec{A}(\vec{r}') \cdot d\vec{r}' + \frac{e}{\hbar}\int_{r_A}^{r_F} \vec{A}(\vec{r}') \cdot d\vec{r}'$$

$$= \frac{e}{\hbar}\oint \vec{A}(\vec{r}') \cdot d\vec{r}'$$



By Stokes' Theorem

$$\frac{e}{\hbar}\oint \vec{A}(\vec{r}') \cdot d\vec{r}' = \frac{e}{\hbar}\Phi$$

The vector potential <u>without a field</u> produces a phase difference which is measurable.

Experimental Results

- Experimental setup identical.
- Here, a’ signifies range of observation.
- Phase shift equal to enclosed flux.

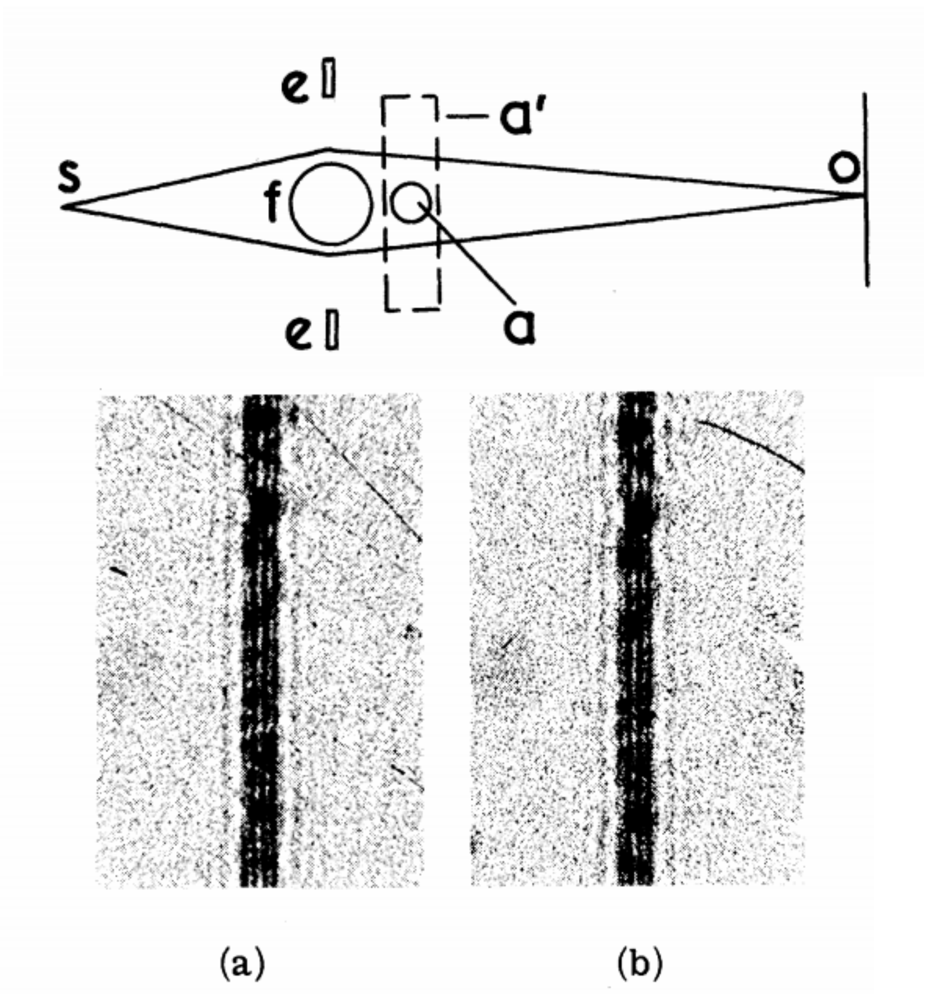


FIG. 2. (a) Fringe pattern due to biprism alone. (b) Pattern displaced by 2.5 fringe widths by field of type a'.

R G Chambers. Shift of an Electron Interference Pattern by Enclosed Magnetic Flux. PRL, 1960.

Berry Phase

- Take a general Hamiltonian $H(x_i; \lambda_j)$
  - x_i are the degrees of freedom (e.g. (x, y, z))
  - λ_j parameters our system depends on (e.g. length L of an infinite potential well)
- Adiabatic theorem lets us slowly vary the parameter to take in the same eigenstate.
- If we take the groundstate and adiabatically vary a parameter and return back to the groundstate,

$$|\psi\rangle \to e^{i\gamma}|\psi\rangle$$

- An overall <u>*measurable*</u> phase difference appears.

Berry Phase

- For a closed contour, this phase is called the **Berry phase** and is computed as

$$\gamma = -\oint \mathcal{A}_i(\lambda) d\lambda_i$$

Where $\mathcal{A}_i(\lambda)$ is the **Berry connection**

$$\mathcal{A}_i(\lambda) = -i\langle\psi| \frac{\partial}{\partial \lambda_i} |\psi\rangle$$

Applying Stokes' Theorem, we can define the **Berry curvature**

$$\mathcal{F}_{ij}(\lambda) = \frac{\partial \mathcal{A}_i}{\partial \lambda_j} - \frac{\partial \mathcal{A}_j}{\partial \lambda_i}$$

Aharonov- Bohm Connection

- In essence, take $\mathcal{A}_i(\lambda) = \vec{A}$ (vector potential) then $\mathcal{F}_{ij}(\lambda) = \vec{\nabla}\times\vec{A}$
- Region without $\vec{B}$ we recover Aharonov- Bohm
- The Aharonov-Bohm is a Berry phase with real fields

Summary

- The phase difference is measurable, not just a phase

$$\psi(x) \sim e^{i\phi}\psi(x), \text{ phase not measurable}$$
$$\psi(x) \rightarrow e^{i\Delta\phi}\psi(x), \text{ phase difference is}$$

- Analogues classical effect called **parallel transport**.
  - Taking a vector around a closed path on a curved surface rotates the vector
- The Aharonov-Bohm Effect shows the vector potential is real
  - Produces measurable effects
- An application of Berry phases
  - Useful notion for studying topological systems