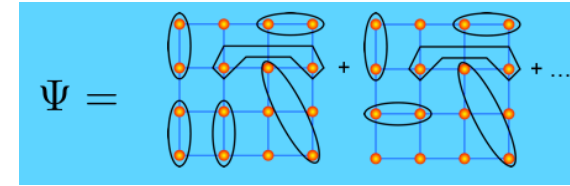


Spin Liquids

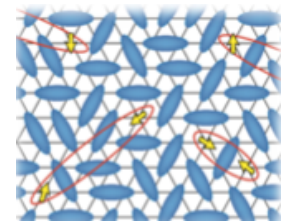
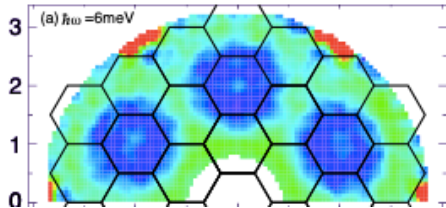
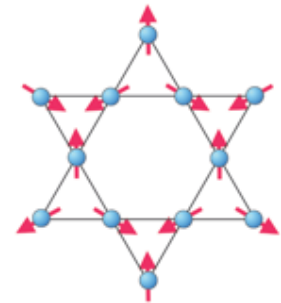
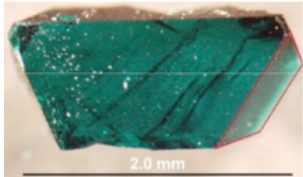


What They are NOT

Exotic Excitations

Wavefunctions

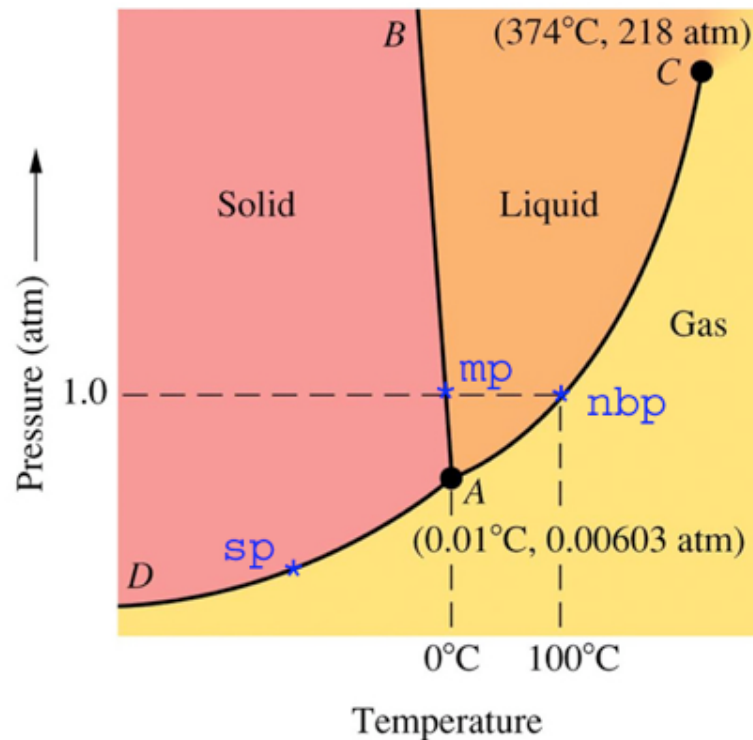
Experimental Candidates



My 15 minute Presentation (again!)
“Intro to Quantum Criticality”

Spin Liquid = Spin Gas ??

- Classical liquids are continuously connected to gases - *not* a distinct phase of matter ...

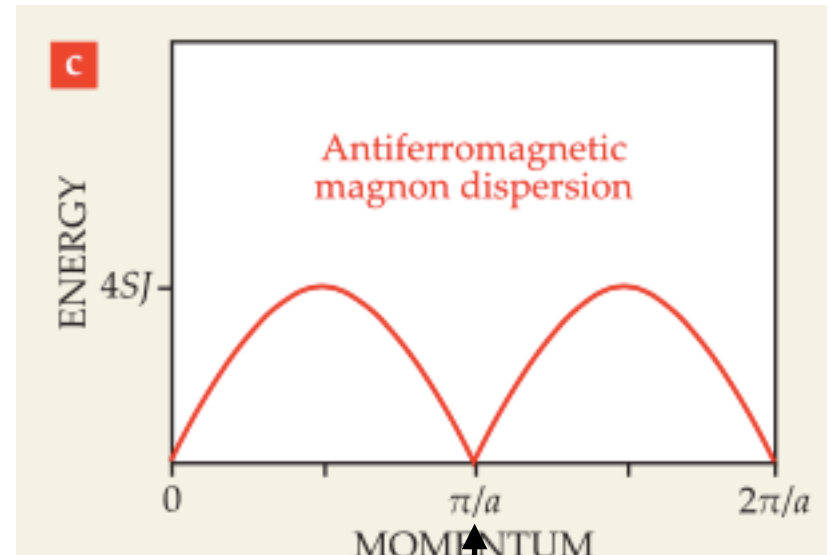
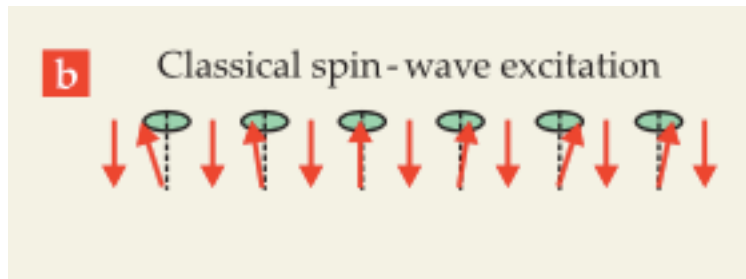


Spin Waves vs. Spinons

Spin Waves

Low-Energy States of an Ordered Magnet

Quantum of Excitation = Magnon



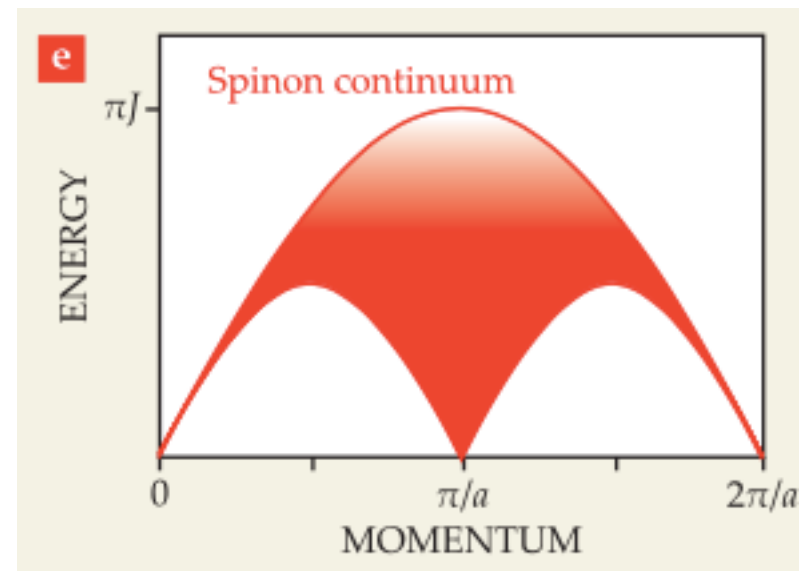
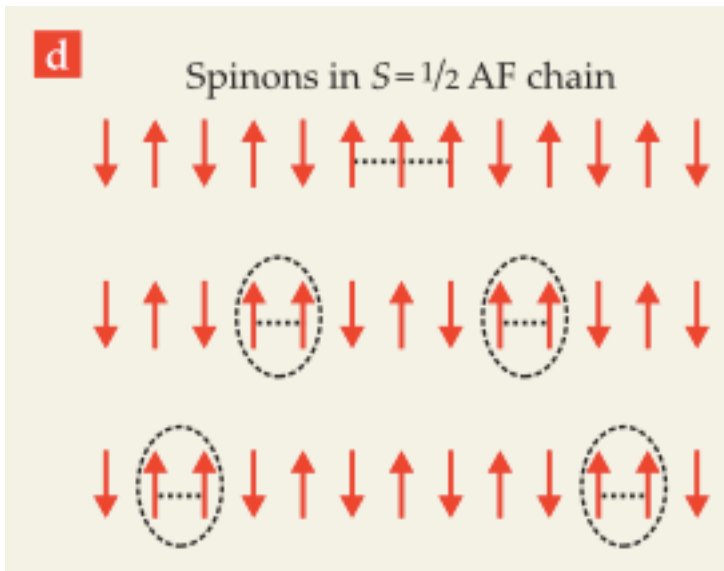
Goldstone Mode

Spin Waves vs. Spinons

Spinons

$S = 1/2$ AFM chain: No Long-Range Order

No Breaking of Spin Rotational Symmetry



$S = 1$ **fractionalized** into two $S=1/2$ spinons !!

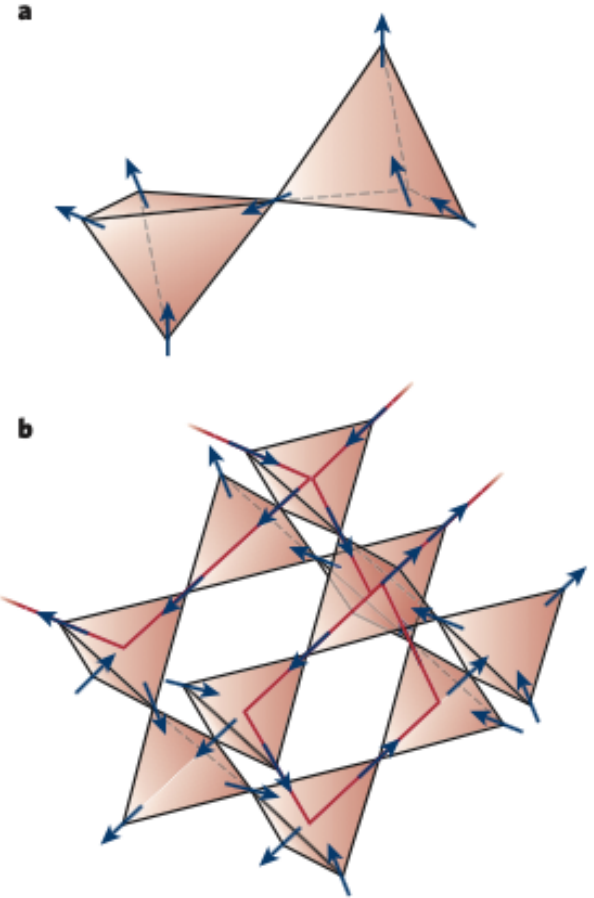
Classical Spin Liquid: Spin Ice at Low T ??

Spin Freezing !!

Large Energy Barriers
between Ice-Rule
Spin States

Weak Quantum Fluctuations

Strong Quantum Fluctuations Needed to
avoid Ordering or Freezing even at $T=0$!!

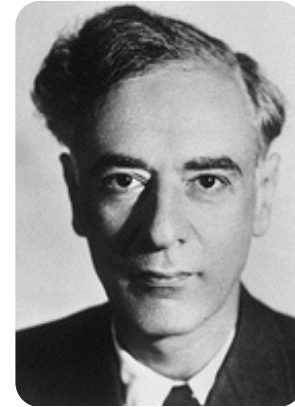


A debate

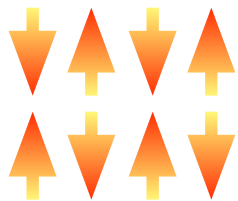


Néel

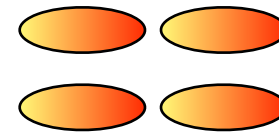
$$H = |J| \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



Landau



antiferromagnet

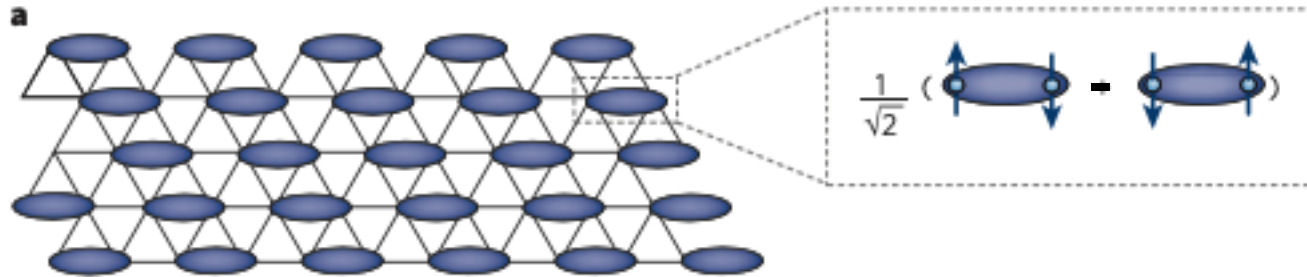


$$\text{orange oval} = \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)$$

singlets

(courtesy: L. Balents)

Valence Bond States of Frustrated Antiferromagnets



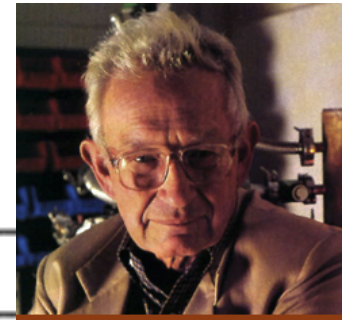
Valence Bond Solid (VBS)

Breaks Lattice Symmetries

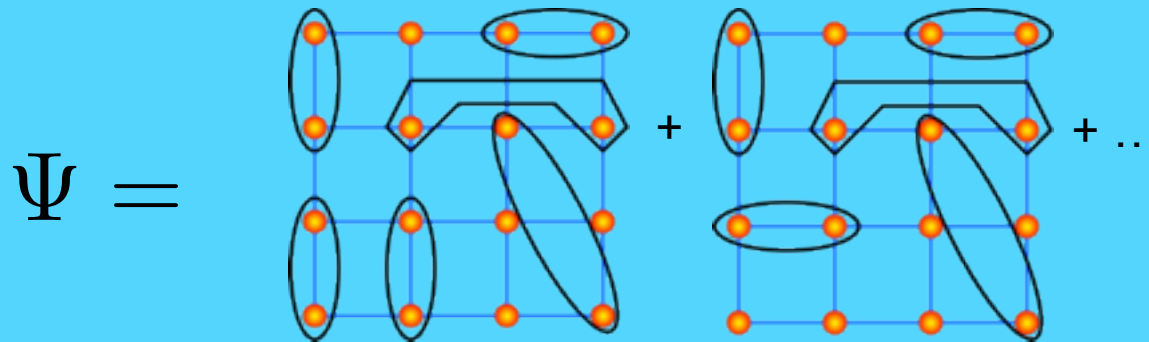
No Long-Range Entanglement

The Resonating Valence Bond State in La_2CuO_4 and Superconductivity

P. W. ANDERSON



Science 87

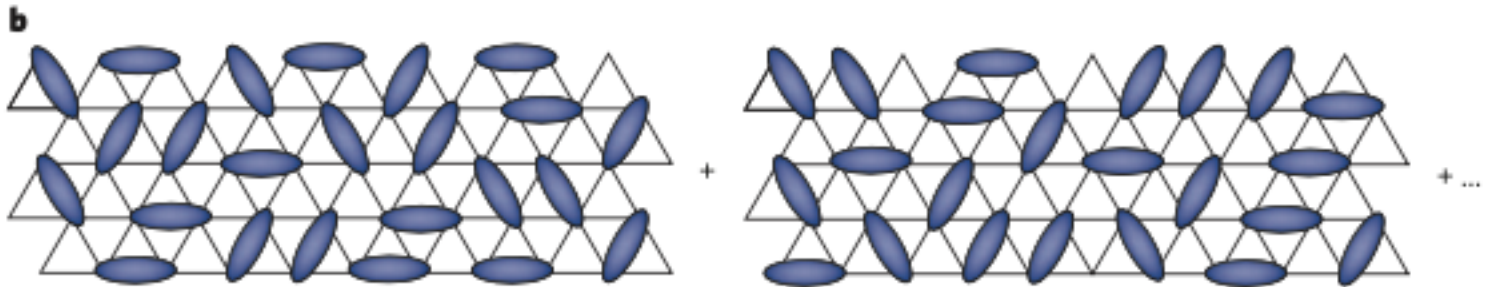


Dearth of 2D $S=1/2$ AFMS: Could Doping such a "Spin Liquid" lead to Novel Superconductivity??

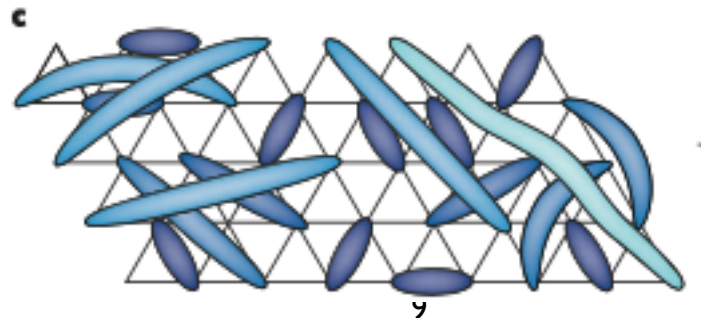
Resonating Valence Bond (RVB) States of Frustrated Antiferromagnets

Wavefunction is superposition of many pairings of states with

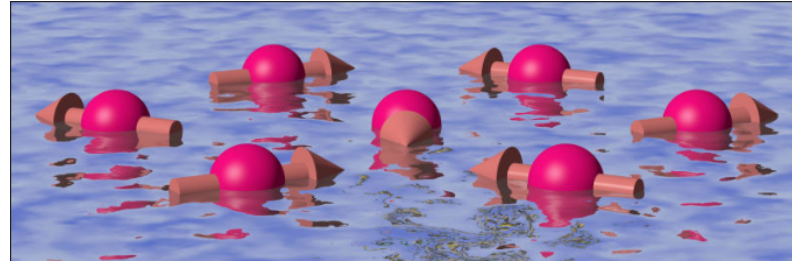
Short-Range Bonds



Longer-Range Bonds



Quantum Spin Liquids:



Quantum Spin System with No Spin Ordering or Spin Freezing

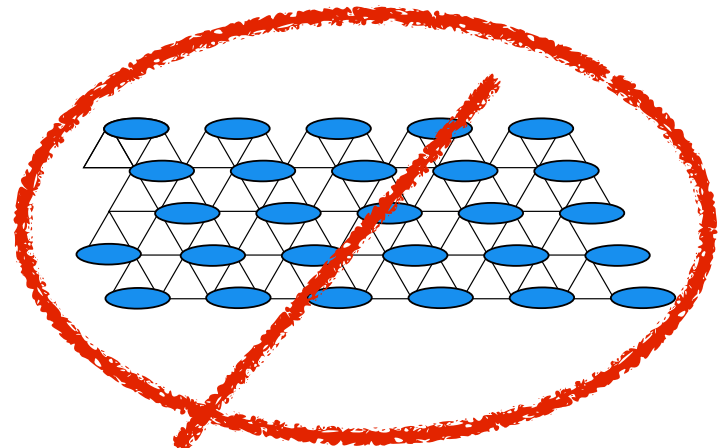
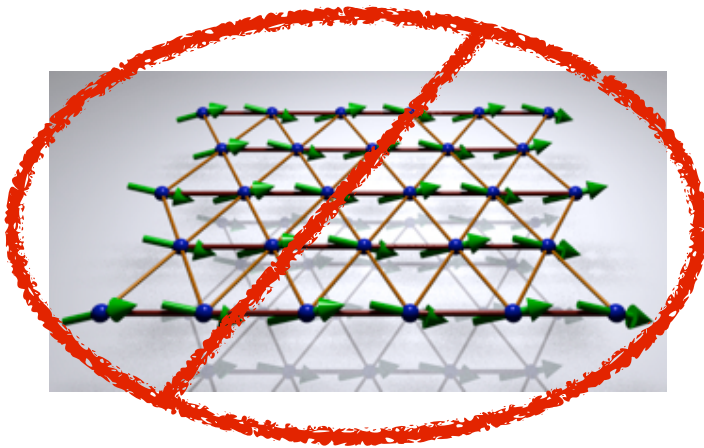
No Broken Symmetry

Expected: Unconventional Excitations (NOT spin 1 magnons!)

“Economy” Strongly Correlated Phases

A Modern View

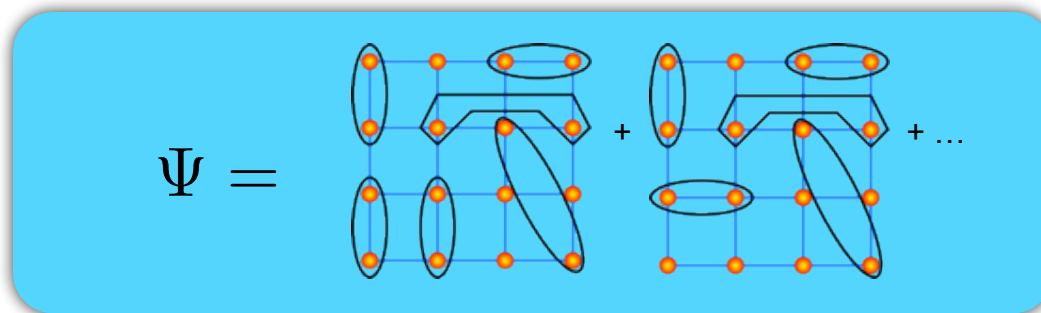
- Let's call a QSL a ground state of a spin system with *long range entanglement*
- This means a state which cannot be regarded or even approximated as a product state over any finite blocks



(courtesy: L. Balents)

A Modern View

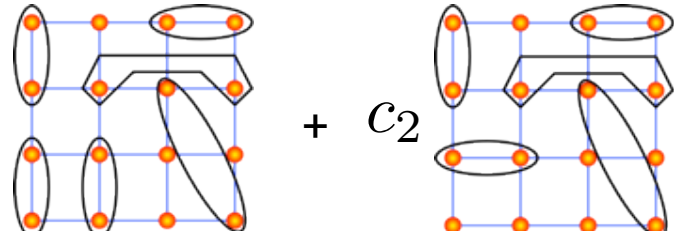
- Let's call a QSL a ground state of a spin system with *long range entanglement*
- This means a state which cannot be regarded or even approximated as a product state over any finite blocks



(courtesy: L. Balents)

How to describe a QSL?

- A long-range entangled wavefunction is a complicated thing!

$$\Psi = c_1 \text{ [Diagram 1] } + c_2 \text{ [Diagram 2] } + \dots$$


- Very hard to work directly with all these coefficients

Quantum Spin Liquid “Zooology”

Characterized by weights of VB partition in Wavefunction

Gapped and Gapless

Excitations obey fermion, bose and anyon statistics

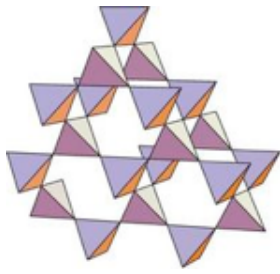
Two common varieties:

U(1) States: unpaired spinons, gapless in 2d,
strong gauge fluctuations, stable only at $T=0$ in $d=3$

Z_2 States: Spinons paired, gapped in 2d,
weak gauge fluctuations, Ising transition in $d=3$

Where to look?

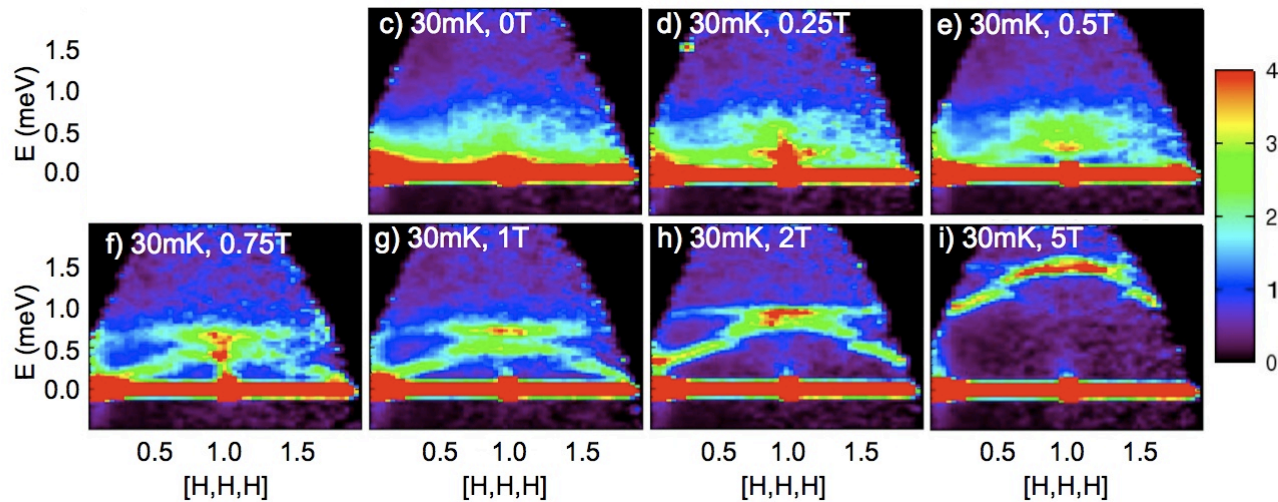
- Materials with
 - $S=1/2$ spins
 - Frustration
 - Significant charge fluctuations
 - Exotic interactions (c.f. Spin-orbit coupling)



Yb₂Ti₂O₇

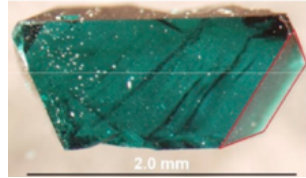
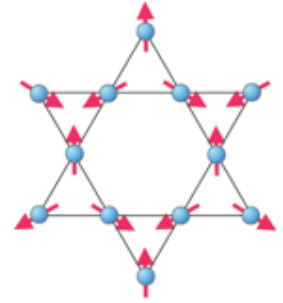
pyrochlore lattice

K.A. Ross *et al* (2009)



- Spin waves appear absent in low field, but emerge for $B > 0.5\text{T}$
- a low field QSL?

Herbertsmithite



$S=1/2$ Kagome Material with $J \sim 200\text{K}$

No conventional magnetic order to 50mK

PRL 103, 237201 (2009)

PHYSICAL REVIEW LETTERS

week ending
4 DECEMBER 2009

Scale-Free Antiferromagnetic Fluctuations in the $s = 1/2$ Kagome Antiferromagnet Herbertsmithite

M. A. de Vries,^{1,2,3,*} J. R. Stewart,^{4,5} P. P. Deen,⁴ J. O. Piatek,² G. J. Nilsen,² H. M. Rønnow,² and A. Harrison^{4,3}

¹*School of Physics & Astronomy, University of St-Andrews, the North Haugh, St Andrews, KY16 9SS, United Kingdom*

²*Laboratory for Quantum Magnetism, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland*

³*CSEC and School of Chemistry, The University of Edinburgh, Edinburgh, EH9 3JZ, United Kingdom*

⁴*Institut Laue-Langevin, 6 rue Jules Horowitz, F-38042 Grenoble, France*

⁵*ISIS facility, Rutherford Appleton Laboratories, STFC, Chilton, Didcot OX11 0DE, United Kingdom*

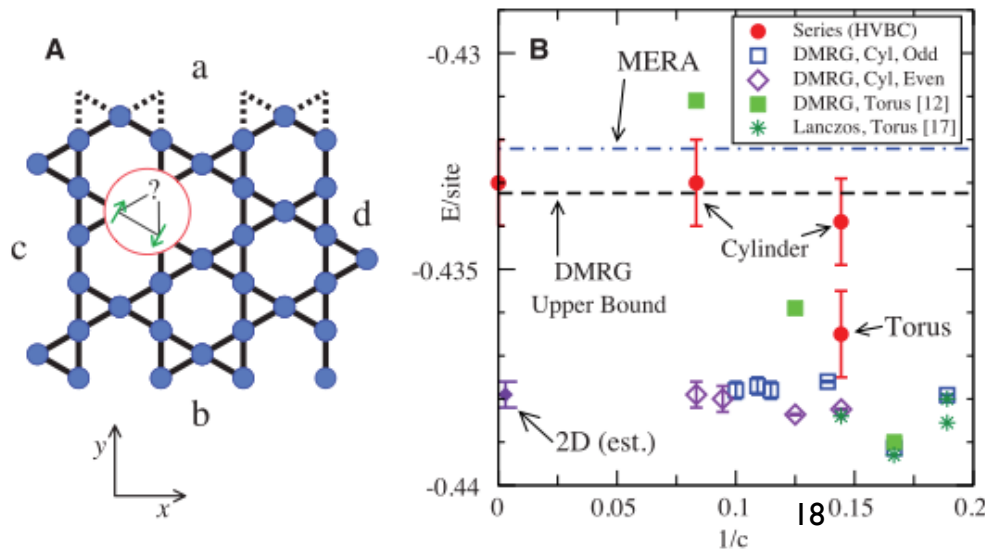
(Received 18 February 2009; revised manuscript received 12 October 2009; published 2 December 2009)

Neutron spectroscopy and diffuse neutron scattering on herbertsmithite [$\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$], a near-ideal realization of the $s = 1/2$ kagome antiferromagnet, reveal the hallmark property of a quantum spin liquid: instantaneous short-ranged antiferromagnetic correlations in the absence of a time-averaged ordered moment. These dynamic antiferromagnetic correlations are weakly dependent of neutron-energy transfer and temperature, and persist up to 25 meV and 120 K. At low energy transfers a shift of the magnetic scattering to low Q is observed with increasing temperature, providing evidence of gapless spinons. It is argued that these observations provide important evidence in favor of resonating-valence-bond theories of (doped) Mott insulators.

Spin-Liquid Ground State of the $S = 1/2$ Kagome Heisenberg Antiferromagnet

Simeng Yan,¹ David A. Huse,^{2,3} Steven R. White^{1*}

We use the density matrix renormalization group to perform accurate calculations of the ground state of the nearest-neighbor quantum spin $S = 1/2$ Heisenberg antiferromagnet on the kagome lattice. We study this model on numerous long cylinders with circumferences up to 12 lattice spacings. Through a combination of very-low-energy and small finite-size effects, our results provide strong evidence that, for the infinite two-dimensional system, the ground state of this model is a fully gapped spin liquid.



Ground-State is Gapped

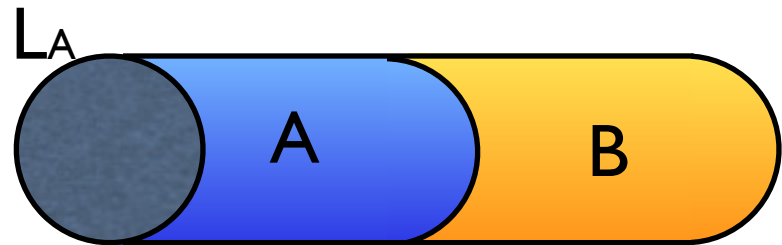
No Sign of Magnetic or Valence Bond Ordering!!

Topological Entanglement Entropy: A New Diagnostic for Phases with No Known Order Parameter

(for gapped Quantum Spin Liquids)

S_A : Measure of how strongly
entangled subregion A is with
rest of the system

$$S_A(L) \sim \alpha L_A - \gamma$$



$$\rho_A = \text{Tr}_B |\psi\rangle\langle\psi|$$

$$S = -\text{Tr}_A [\rho_A \ln \rho_A]$$

Nature of the Spin-Liquid Ground State of the $S = 1/2$ Heisenberg Model on the Kagome LatticeStefan Depenbrock,^{1,*} Ian P. McCulloch,² and Ulrich Schollwöck¹¹Department of Physics and Arnold Sommerfeld Center for Theoretical Physics,
Ludwig-Maximilians-Universität München, 80333 München, Germany²Centre for Engineered Quantum Systems, School of Mathematics and Physics, The University of Queensland,
St. Lucia, Queensland 4072, Australia

(Received 22 May 2012; published 7 August 2012)

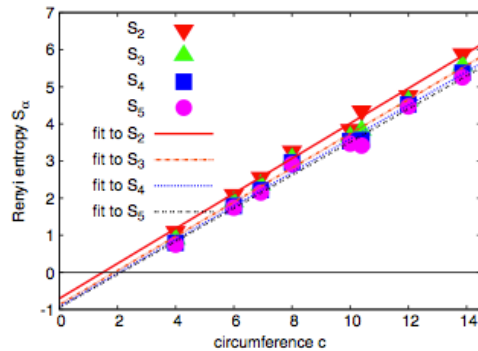
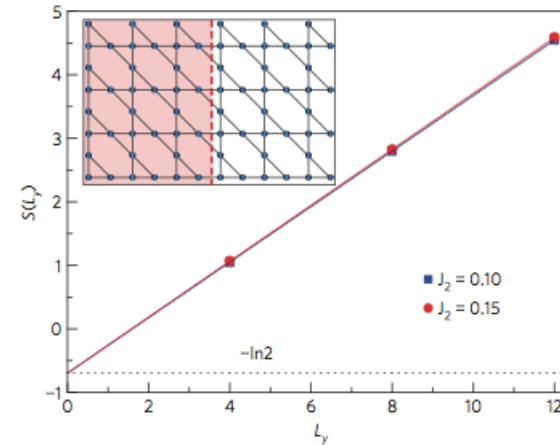


FIG. 6 (color online). Renyi entropies S_α of infinitely long cylinders for various α versus circumference c , extrapolated to $c = 0$. The negative intercept is the topological entanglement entropy γ .

Identifying topological order by entanglement entropy

Hong-Chen Jiang¹, Zhenghan Wang² and Leon Balents^{1*}

$$\Upsilon_{\text{DMRG}} = 0.698(8)$$

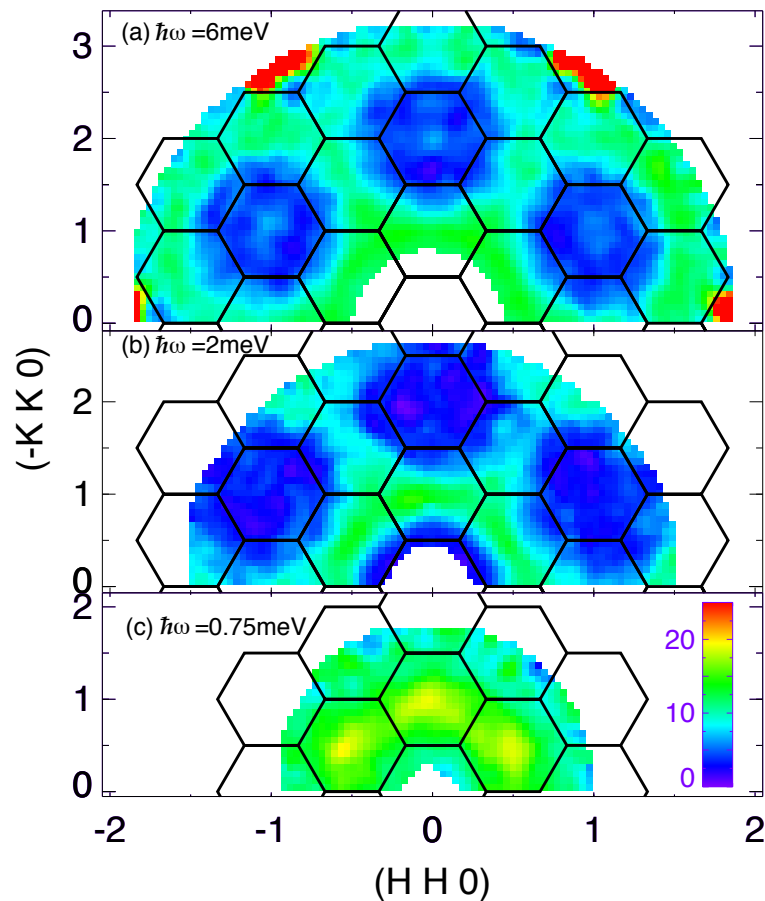
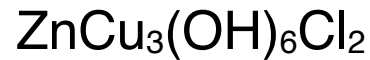
$$(\Upsilon_{\text{th}} = \ln(2) = 0.693)$$

Z_2 Spin Liquid ???!

Fractionalized excitations in the spin-liquid state of a kagome-lattice antiferromagnet

Tian-Heng Han¹, Joel S. Helton², Shaoyan Chu³, Daniel G. Nocera⁴, Jose A. Rodriguez-Rivera^{2,5}, Collin Broholm^{2,6} & Young S. Lee¹

406 | NATURE | VOL 492 | 20/27 DECEMBER 2012



Continuum of Spinon
Excitations !!

No Spin Gap ??

(Disorder) ??

1T-TaS₂ as a quantum spin liquid

K. T. Law^a and Patrick A. Lee^{b,1}

^aDepartment of Physics, Hong Kong University of Science and Technology, Hong Kong, China; and ^bDepartment of Physics, Massachusetts Institute of Technology, Cambridge MA 02139

Contributed by Patrick A. Lee, May 26, 2017 (sent for review April 24, 2017; reviewed by Steven A. Kivelson and N. Phuan Ong)

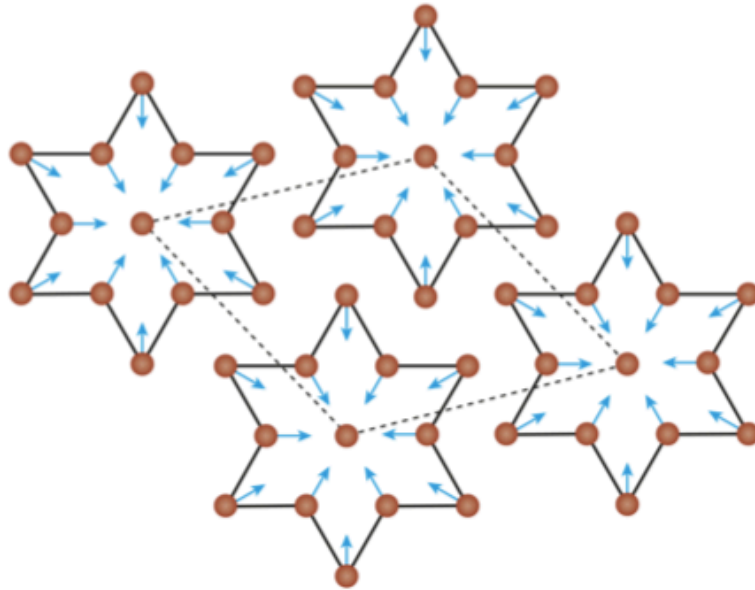


Fig. 1. In the cluster Mott phase of 1T-TaS₂, Ta atoms (red dots) belonging to a star of David move toward the Ta atom at the center. Thirteen Ta atoms form a unit cell, and these unit cells form a triangular lattice. The directions and lengths of the arrows are schematic.

1T-TaS₂ as a quantum spin liquid

K. T. Law^a and Patrick A. Lee^{b,1}

^aDepartment of Physics, Hong Kong University of Science and Technology, Hong Kong, China; and ^bDepartment of Physics, Massachusetts Institute of Technology, Cambridge MA 02139

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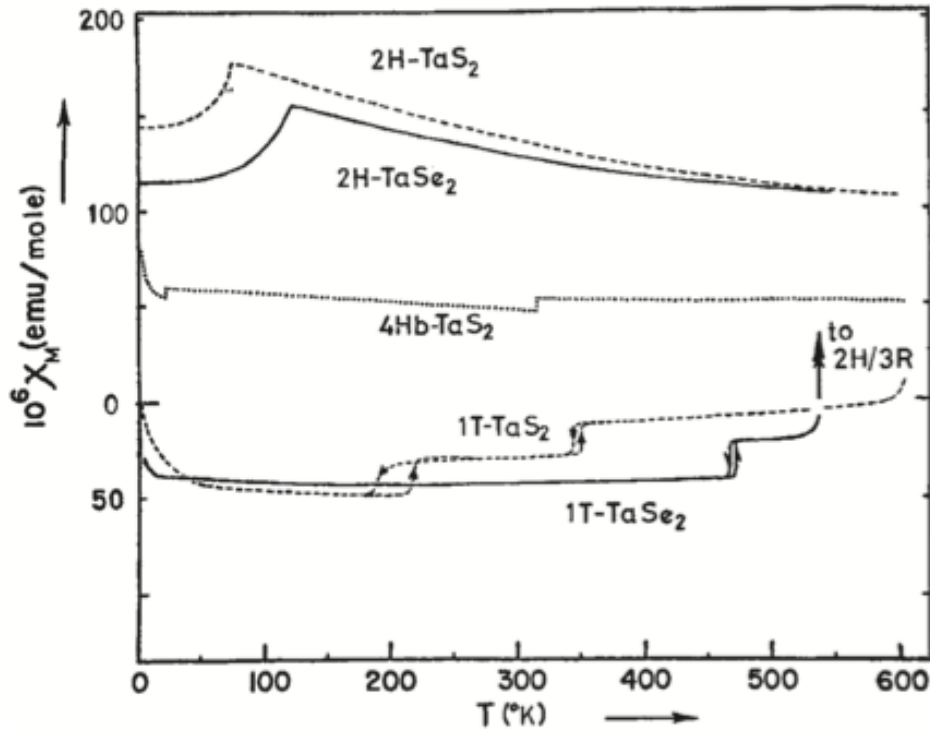


Fig. 2. The molar magnetic susceptibility (χ_M) versus temperature (T) for TaS₂ and TaSe₂ with different lattice structures. The background diamagnetic term has not been subtracted. The data are taken from ref. 3.

Effect of quenched disorder on the quantum spin liquid state of the triangular-lattice antiferromagnet 1T-TaS₂

H. Murayama,¹ Y. Sato,¹ T. Taniguchi,¹ R. Kurihara,¹ X. Z. Xing,¹ W. Huang,¹ S. Kasahara,¹ Y. Kasahara,¹ I. Kimchi,² M. Yoshida,³ Y. Iwasa,^{3,4} Y. Mizukami,⁵ T. Shibauchi,⁵ M. Konczykowski,⁶ and Y. Matsuda¹

¹*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

²*JILA, NIST, and Department of Physics, University of Colorado, Boulder, Colorado, USA*

³*RIKEN Center for Emergent Matter Science (CEMS), Wako 351-0198, Japan*

⁴*Quantum-Phase Electronics Center and Department of Applied Physics, University of Tokyo, Tokyo 113-8656, Japan*

⁵*Department of Advanced Materials Science, University of Tokyo, Chiba 277-8561, Japan*

⁶*Laboratoire des Solides irradiées, CEA/DRF/IRAMIS, Ecole Polytechnique, CNRS, Institut Polytechnique de Paris, F-91128 Palaiseau, France*

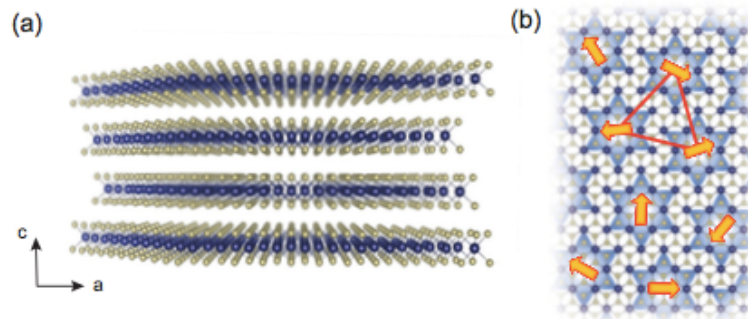


FIG. 1. (a) Crystal structure of 1T-TaS₂. Each layer consists of tantalum atoms (purple) sandwiched by sulfur atoms (yellow). (b) A schematics of the star-of-David clusters, which appear as a result of the C-CDW transition. In the Mott insulating state, the electrons localized at the centers of the clusters form $S = 1/2$ 2D perfect triangular lattice.

Effect of quenched disorder on the quantum spin liquid state of the triangular-lattice antiferromagnet $1T\text{-TaS}_2$

H. Murayama,¹ Y. Sato,¹ T. Taniguchi,¹ R. Kurihara,¹ X. Z. Xing,¹ W. Huang,¹ S. Kasahara,¹ Y. Kasahara,¹ I. Kimchi,² M. Yoshida,³ Y. Iwasa,^{3,4} Y. Mizukami,⁵ T. Shibauchi,⁵ M. Konczykowski,⁶ and Y. Matsuda¹

¹*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

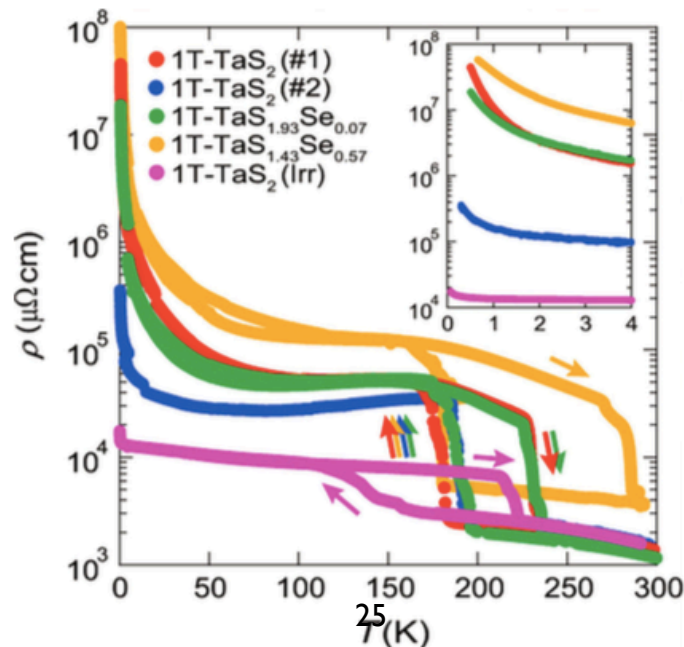
²*JILA, NIST, and Department of Physics, University of Colorado, Boulder, Colorado, USA*

³*RIKEN Center for Emergent Matter Science (CEMS), Wako 351-0198, Japan*

⁴*Quantum-Phase Electronics Center and Department of Applied Physics, University of Tokyo, Tokyo 113-8656, Japan*

⁵*Department of Advanced Materials Science, University of Tokyo, Chiba 277-8561, Japan*

⁶*Laboratoire des Solides Irradiés, CEA/DRF/IRAMIS, Ecole Polytechnique, CNRS, Institut Polytechnique de Paris, F-91128 Palaiseau, France*



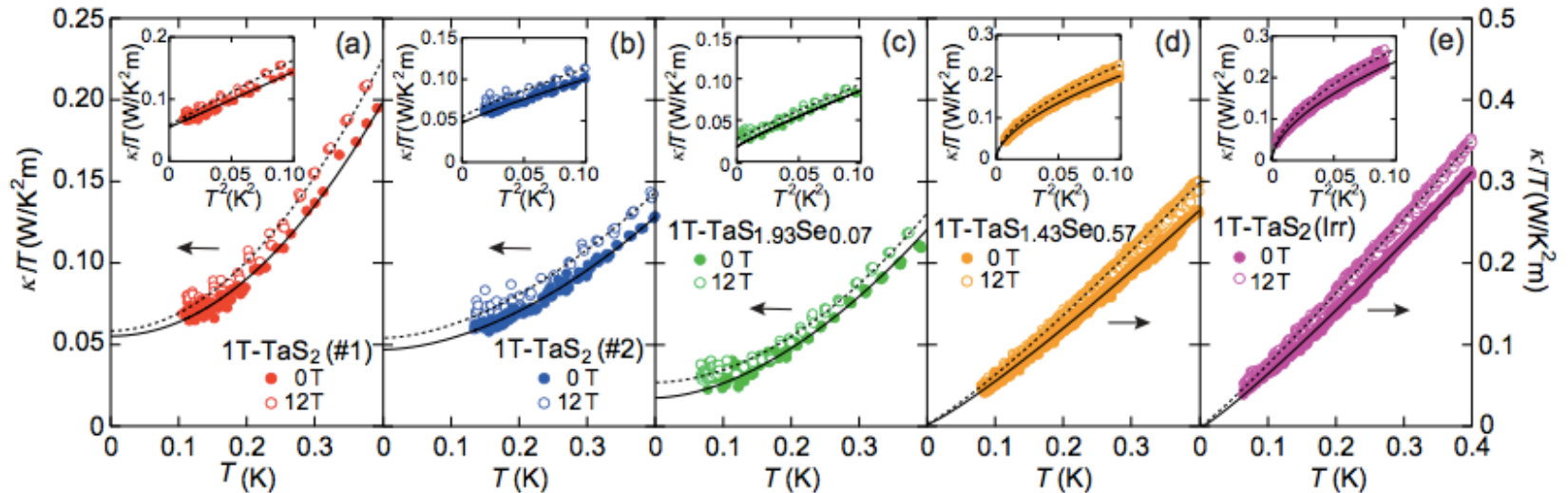


FIG. 3. Temperature dependences of κ/T for (a) 1T-TaS₂ #1, (b) 1T-TaS₂ #2, (c) 1T-TaS_{1.93}Se_{0.07}, (d) 1T-TaS_{1.43}Se_{0.57}, and (e) 1T-TaS₂ (Irr) in zero magnetic field (filled circles) and at $\mu_0 H = 12$ T applied along c axis (open circles). The insets show κ/T plotted as a function of T^2 . Solid and dotted lines in panels (a)–(c) show the fits by the formula $\kappa/T = \kappa_0/T + b_1 T^2$ for 0 and 12 T, respectively. Solid and dotted lines in panels (d) and (f) show the fits by $\kappa/T = \kappa_0/T + b_2 T^p$ with $p \approx 1$ for 0 T and 12 T, respectively. The residual value κ_0/T (≈ 0.05 W/K²m) for 1T-TaS₂ #1 and #2 is largely suppressed in 1T-TaS_{1.93}Se_{0.07} ($\kappa_0/T \approx 0.02$ W/K²m). In 1T-TaS_{1.43}Se_{0.57} and 1T-TaS₂ (Irr), κ_0/T is absent.

Effect of quenched disorder on the quantum spin liquid state of the triangular-lattice antiferromagnet $1T\text{-TaS}_2$

H. Murayama,¹ Y. Sato,¹ T. Taniguchi,¹ R. Kurihara,¹ X. Z. Xing,¹ W. Huang,¹ S. Kasahara,¹ Y. Kasahara,¹ I. Kimchi,² M. Yoshida,³ Y. Iwasa,^{3,4} Y. Mizukami,⁵ T. Shibauchi,⁵ M. Konczykowski,⁶ and Y. Matsuda¹

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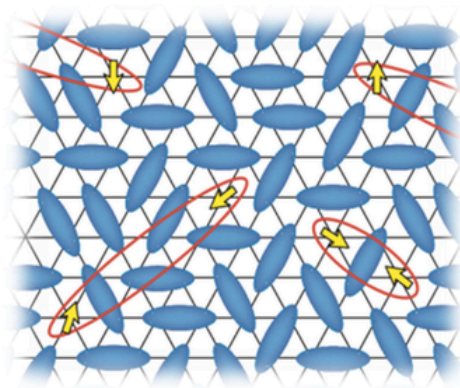
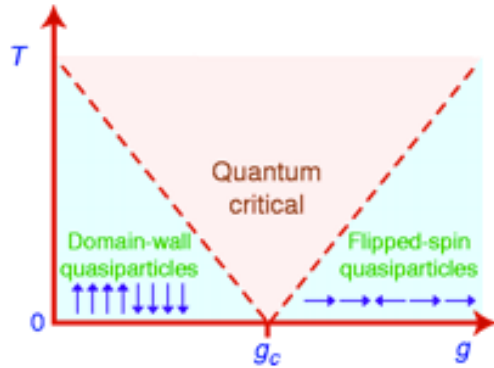


FIG. 7. A schematic illustration of the proposed spin liquid state of $1T\text{-TaS}_2$, which contains gapless excitations with localized and itinerant characters. The localized excitations arise from orphan spins (yellow arrows) that form random long-range valence bonds (red ellipsoids). They are surrounded by the proposed spin liquid state (blue ellipsoids) with itinerant excitations, which may be attributed to spinons that form a Fermi surface.

Introduction to Quantum Criticality

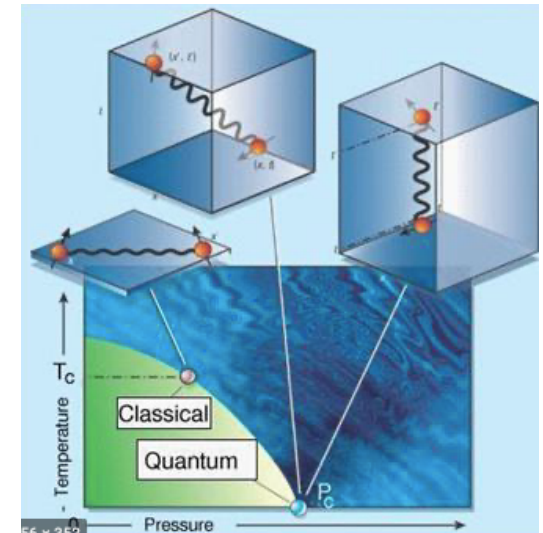


Quantum Criticality ??

Quantum Fluctuations at Finite T ??

An Experimental Case

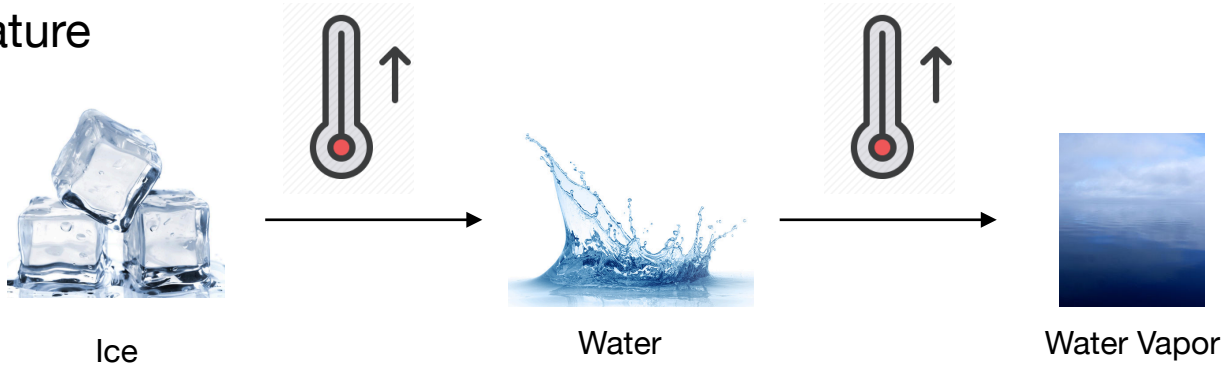
Towards the Future....



Thermal vs. Quantum Fluctuations

Temperature

$$k_B T$$



Heat \longrightarrow Atomic Motion

Thermal Transformation of Matter

Zero-Point Energy

$$\hbar \Omega$$

Heisenberg Uncertainty Principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Atoms Still Moving at Very Low Temperatures

Helium Doesn't Freeze

Superfluidity



Quantum Fluctuations at Finite Temperature ??

Heisenberg Uncertainty Principle

$$\Delta t \propto \frac{\hbar}{\Delta E}$$

Decoherence Time-Scale (Planck time)

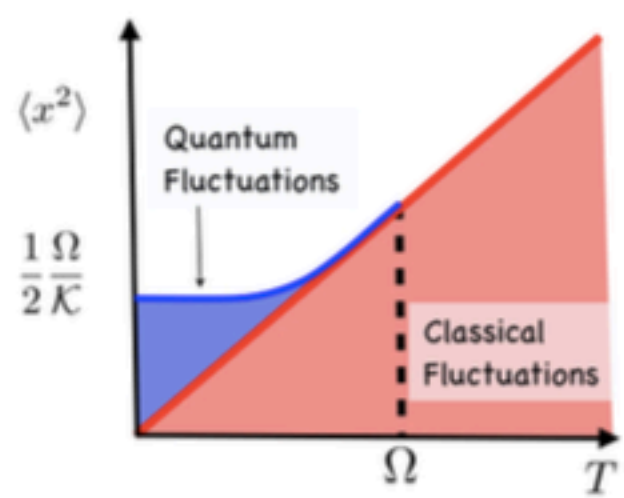
$$t_P \propto \frac{\hbar}{k_B T}$$

Fluctuations purely Quantum up to the Planck time
Classical beyond

T = 0 Quantum Critical Point,
Fluctuations are Purely Quantum

Simple Harmonic Oscillator

Variance $\langle x^2 \rangle = \frac{\Omega}{\mathcal{K}} \left\{ \frac{1}{e^{\frac{\Omega}{T}} - 1} + \frac{1}{2} \right\} \quad (\hbar = 1, k_B = 1)$



$\Omega < T$ Thermal (classical) Fluctuations

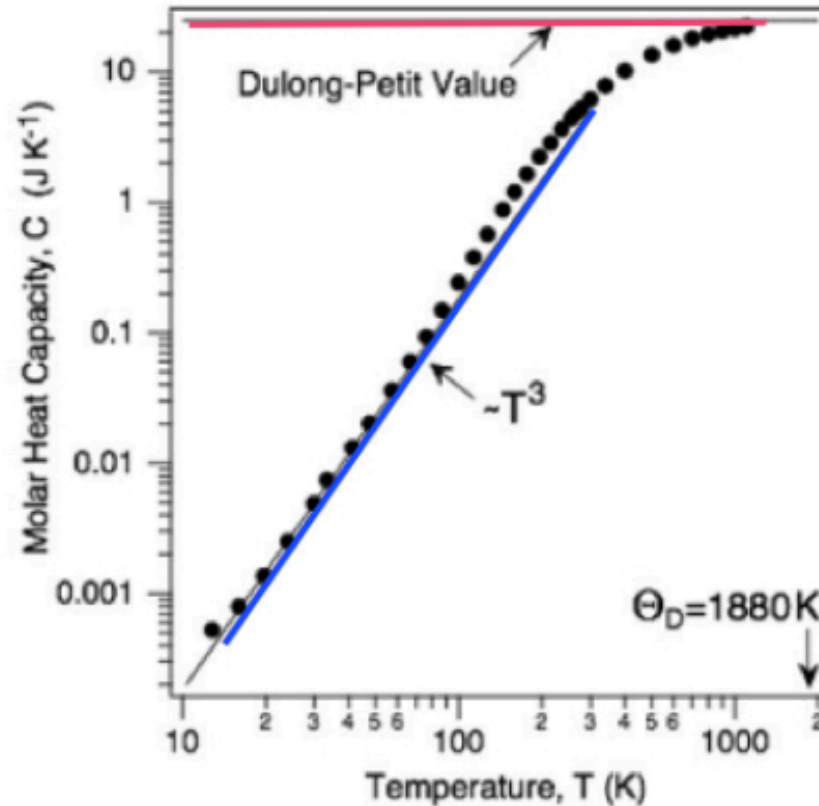
$$\langle x^2 \rangle \sim \frac{T}{\mathcal{K}}$$

$0 < T < \Omega$ Thermal-Quantum Fluctuations

$T = 0$ Pure Quantum Fluctuations

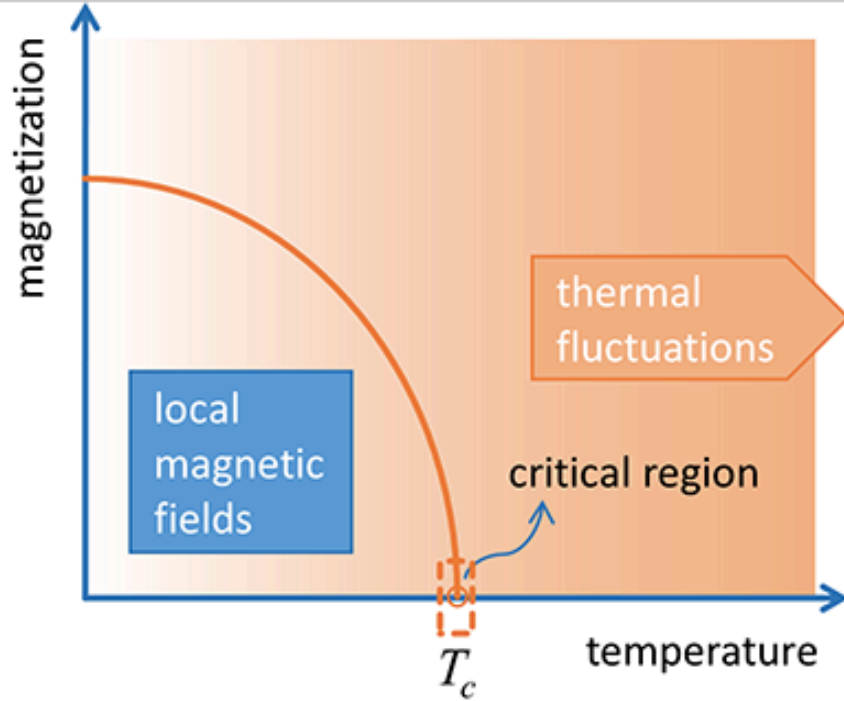
$$\langle x^2 \rangle = \frac{\Omega}{2\mathcal{K}}$$

Specific Heat of Diamond

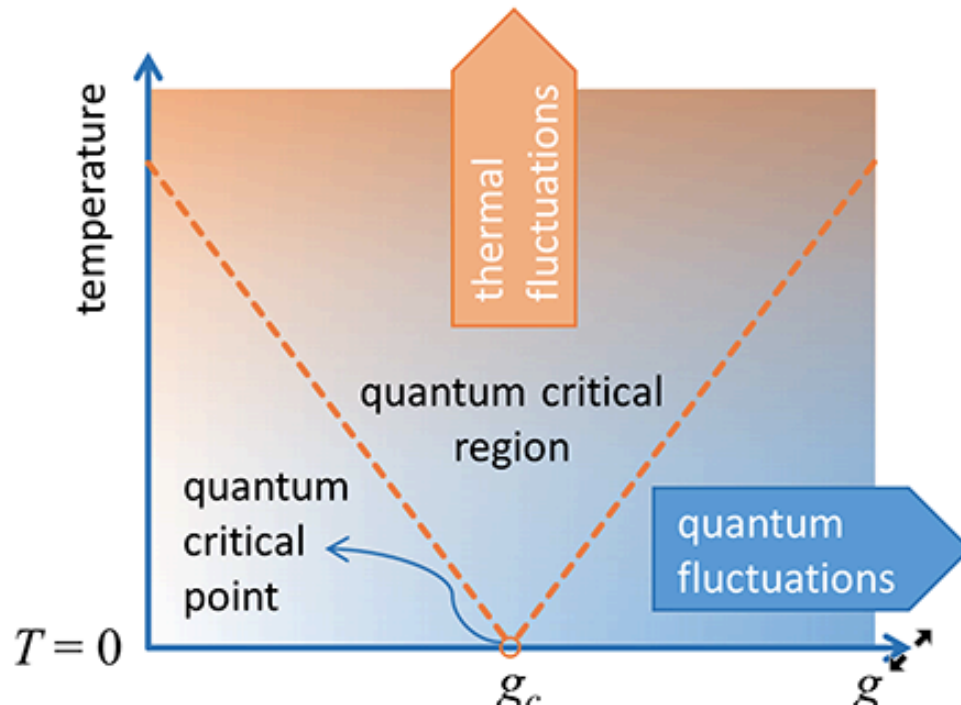


Quantum Fluctuations Present at Room Temperatures !!

Classical Criticality



Quantum Criticality



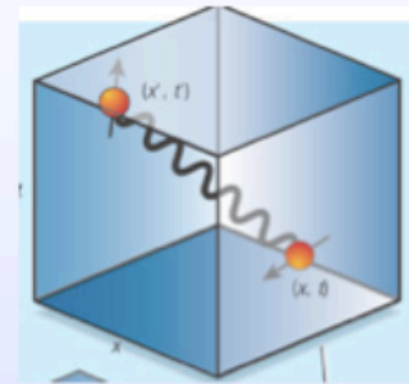
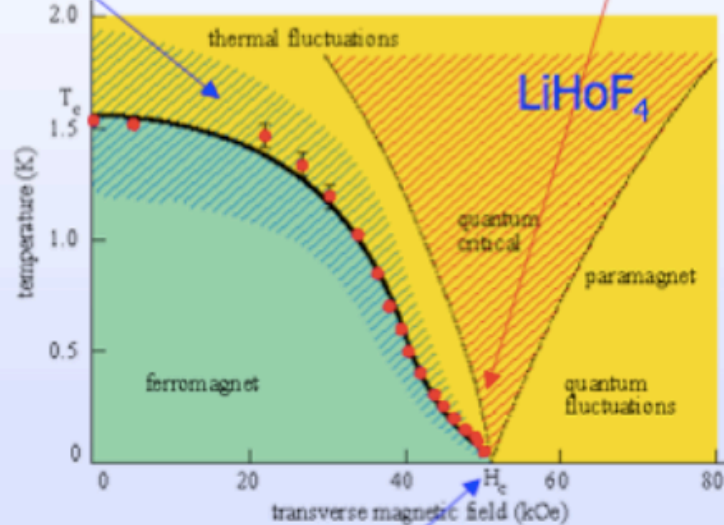
Classical and Quantum Phase Transitions

classical phase transition

$$\xi \propto (T - T_c)^{-\nu_c} \quad (T \gg J)$$

quantum critical regime

$$\xi(T) \propto T^{-\frac{1}{z}} \quad (T \ll J)$$



$$d_{eff} = d + z$$

D. Bitko et al., Phys. Rev. Lett. 77, 940 (1996)

2nd order quantum phase transition

$$\xi(T = 0) \propto (g - g_c)^{-\nu}$$

Towards the Future

Universality of Quantum Criticality ??

Exotic Quantum Phases near QCPs ??

New Functionalities ??



