2/11 Spin Liquids

What They Are NOT

Systems of Interacting Spins with

NO spin ordering

NO broken symmetries

No spin ordering — space / — time

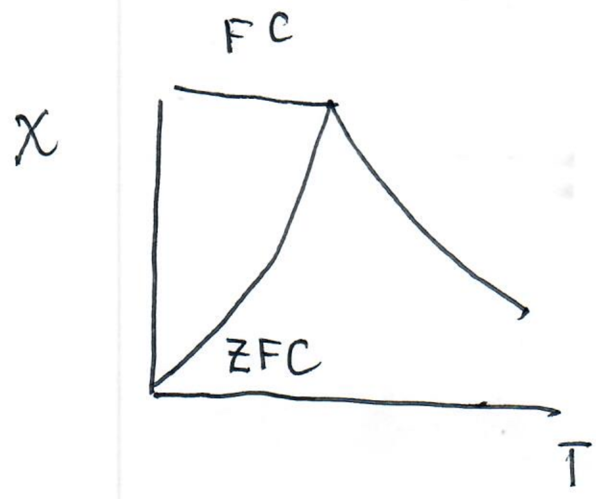
↑ No spin freezing

$$q_{EA} = \lim_{t \to \infty} \lim_{N \to \infty} \left[\langle S_i(t_0)\, S_i(t_0 + t) \rangle \right]$$

average over long set of t_0's

$= 0$ ergodic

$\neq 0$ system "trapped" in single phase



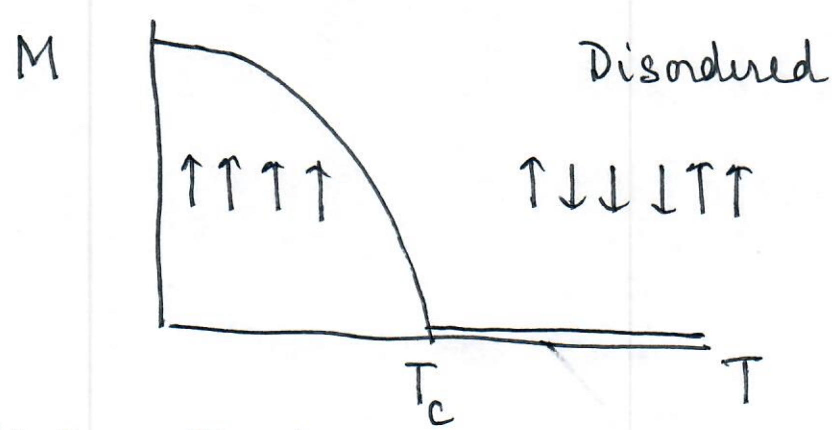
History-dependance!

Absence of magnetic ordering of any type!

NOT described by Landau theory

Brief Discussion of Landau theory

Let's go back to an Ising FM



$M \neq 0 \quad T < T_c$

$M = 0 \quad T > T_c$

M = "order parameter"

Landau theory = effective theory of
an order parameter

Key assumptions

- at fixed value of the order parameter M, $f(M)$ is analytic $\left(f = \frac{F}{V}\right)$

(non-analyticity at T_c occurs because in partition function must solve over all values of M)

Ising model

$$Z = \sum_{(\sigma_j = \pm 1)} e^{-\beta E}$$

Let $M = \sum_j \frac{\sigma_j}{N}$ for N sites.

$$Z = \sum_{M} \sum_{\left(\sigma_j = \pm 1 \,\middle|\, \sum_j \sigma = MN\right)} e^{-\beta E}$$

$$e^{-\beta V f(M)} \equiv \sum_{\left(\sigma_j = \pm 1 \,\middle|\, \sum_j \sigma = MN\right)} e^{-\beta E}$$

where V = volume

For large N, M is essentially continuous

$$\Downarrow$$

$$Z = \int_{-1}^{+1} dM \, e^{-\beta V f(M)} .$$

Landau's assumptions near T_c

- $f(M)$ analytic $\Rightarrow$ can be expressed as Taylor expansion near critical point
- $f(M)$ obeys symmetries of H

$$f(M) - f_0 = \frac{\tilde{\alpha}}{2}(T)\, M^2 + \frac{\tilde{\beta}}{4} M^4$$

$M \to -M$ (no field) $\Rightarrow$ only even powers of M

$$\tilde{\alpha}(T) = \alpha\,(T - T_c)$$

$\tilde{\beta} > 0$ stability $\quad \tilde{\beta} = \beta$

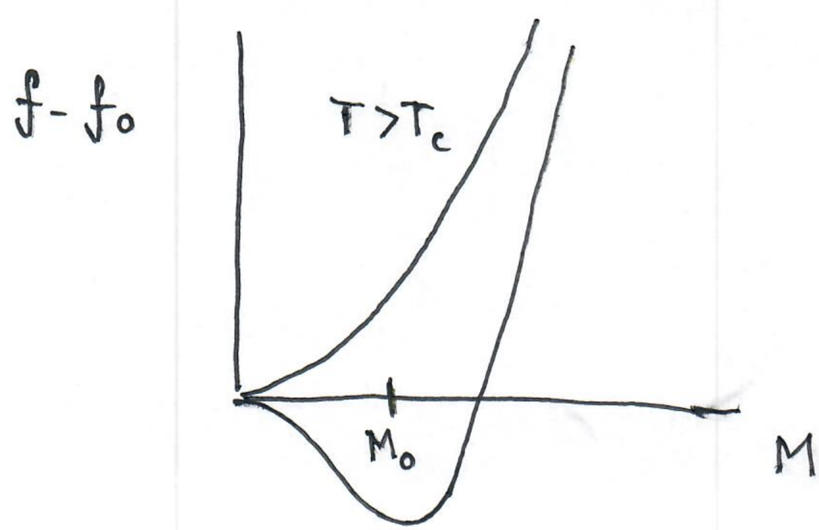
$$\frac{\partial f}{\partial M} = 0 \Rightarrow \alpha(T-T_c) + \beta M^3 = 0$$

$$\Downarrow$$

$$M^2 = -\frac{\alpha(T-T_c)}{\beta}$$

$$M(T) \propto |T-T_c|^{1/2}$$

$$f - f_0 = \begin{cases} -\dfrac{\alpha^2}{2\beta}(T-T_c)^2 & T < T_c \\ 0 & T > 0. \end{cases}$$



Spin Liquids

No Spin Ordering

No Broken Symmetry

No Landau discription

Non-magnetic ground-state built from well-formed local moments

most likely with

Exotic Excitations

Exotic ?

In most phases of matter, the excitations can be constructed from elementary excitations that are either electron-like ($s = 1/2$, $q = \pm e$) or magnon-like (spin $S = 1$, $q = 0$)

standard spin flip excitations are integer changes of the total spin

Fractional?

e.g. Half-integer "spinons"
connected by "tension-free"
strings even at $T = 0$
(emergent gauge theory takes care
of global spin constraint)

Let's return to the 1D AFM
(1d "spin liquid" due to Bethe)
as setting to illustrate spinons.

Classical SLs very difficult to find:

- Order by Disorder
- Spin Freezing

Quantum Mechanics Needed!

Key aspect of Quantum Mechanics:

Superposition

Any linear combination of allowed quantum states

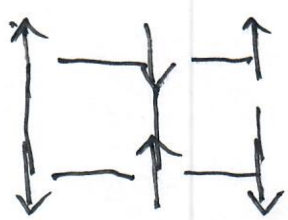
$\Downarrow$

allowed state

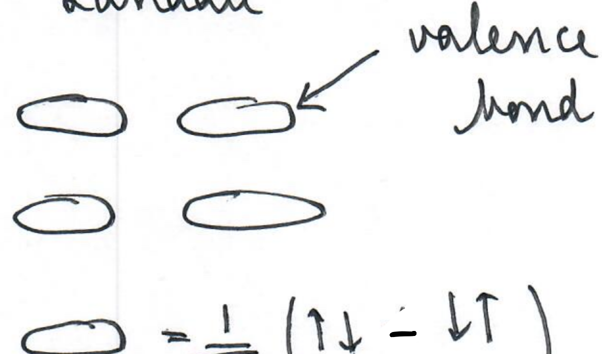
(extended to many electrons).

Recall Néel vs Landau approach to AFM

Néel



Landau



$$= \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)$$

$$H = |J| \sum_{\langle ij \rangle} \vec{S}_i \vec{S}_j .$$

valence bond = entangled pair of spins

1973 Anderson : Dearth of 2D $S = 1/2$ AFMs

Resonating Valence Bond ground-state !

(superposition of all valence bond states)

In valence bond spins are <u>entangled</u> .

Quantum entanglement of spins :

Quantum state of each spin in the group cannot be described independently of the state of the others in the group, even if they are separately by large distances.

<u>Primary feature of Quantum Mechanics</u>