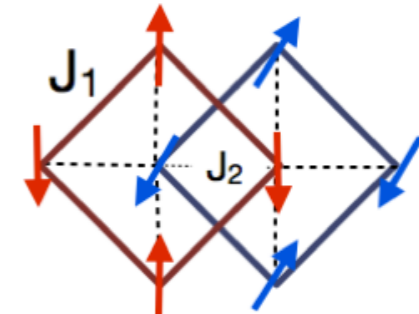


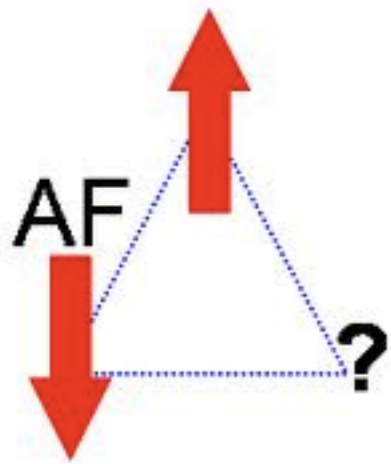
# Spin Models = “Economy” Strongly Correlated Systems



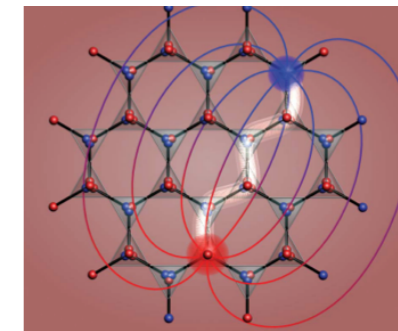
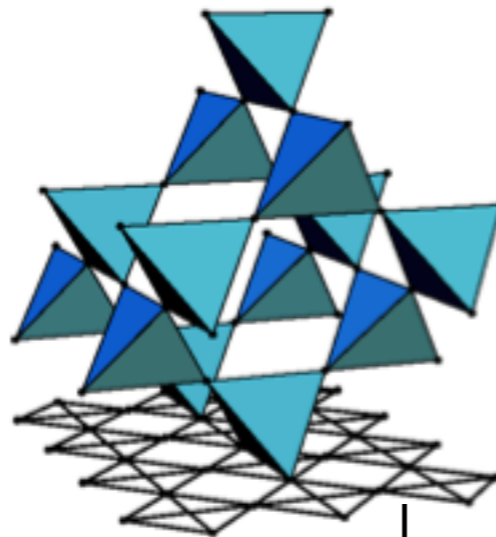
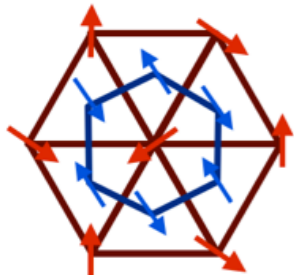
Frustration  $\longrightarrow$  Ordering



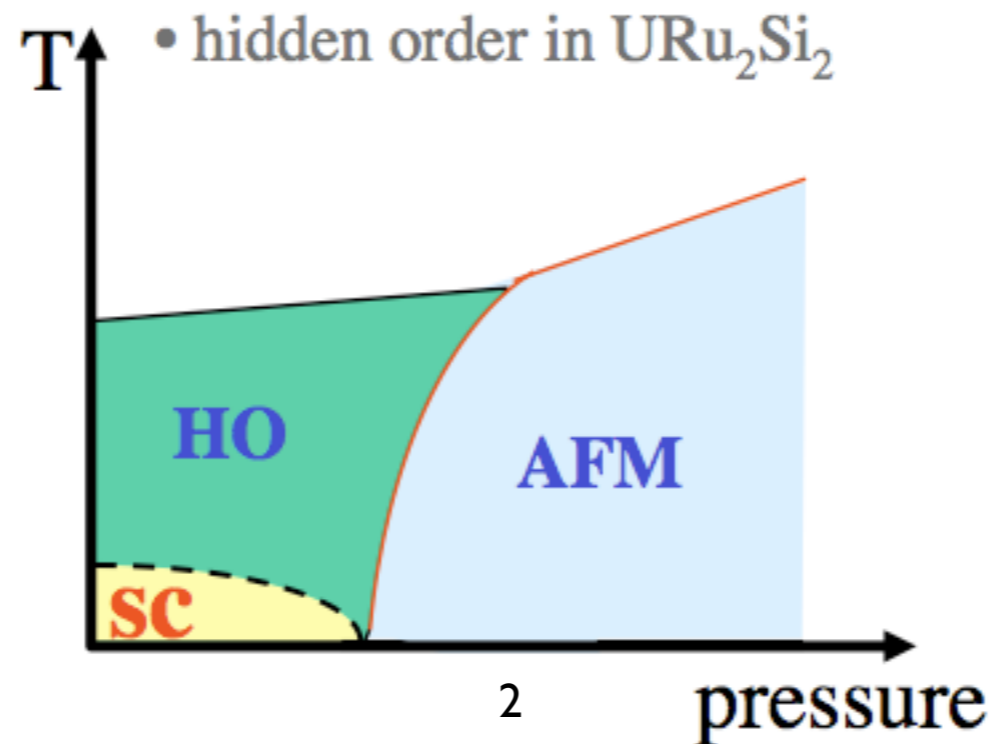
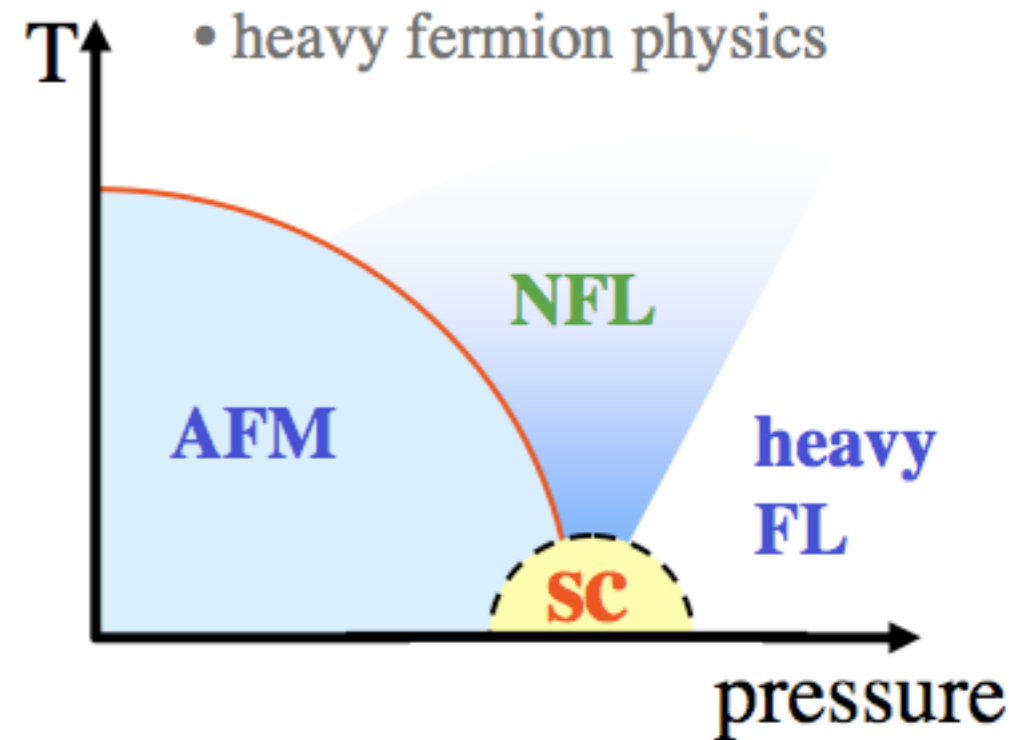
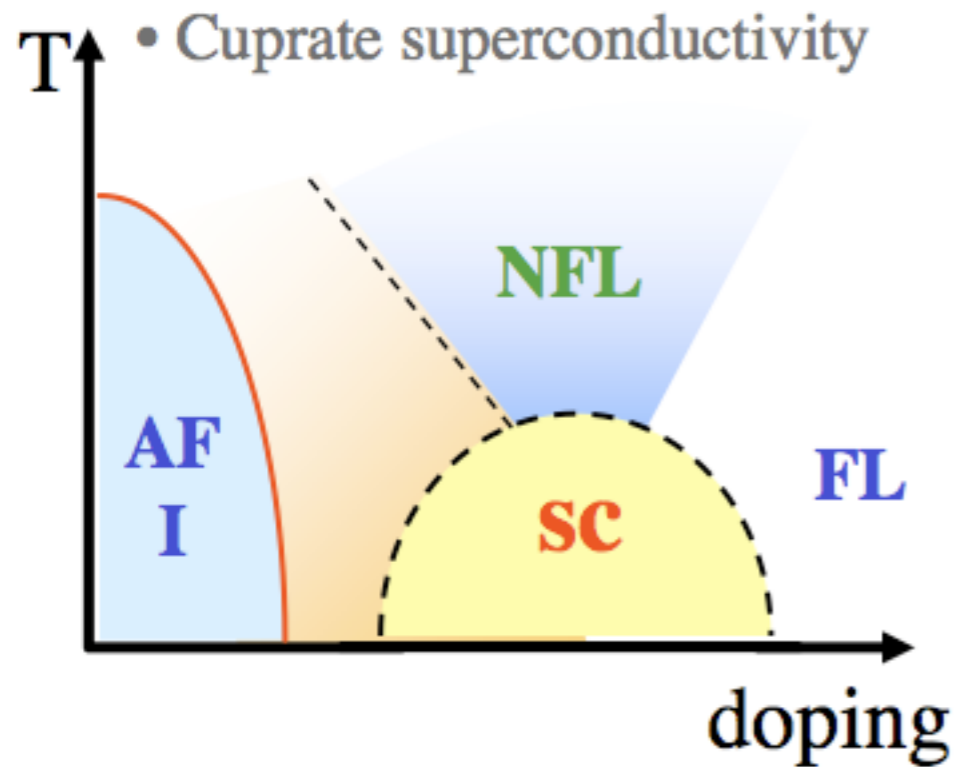
Spin Ice, Fractionalization and Topology



My Fifteen Minute Presentation:  
“Quantum Annealed Criticality”



# Motivation: Novel Materials with Competing Interactions and Exotic Phases



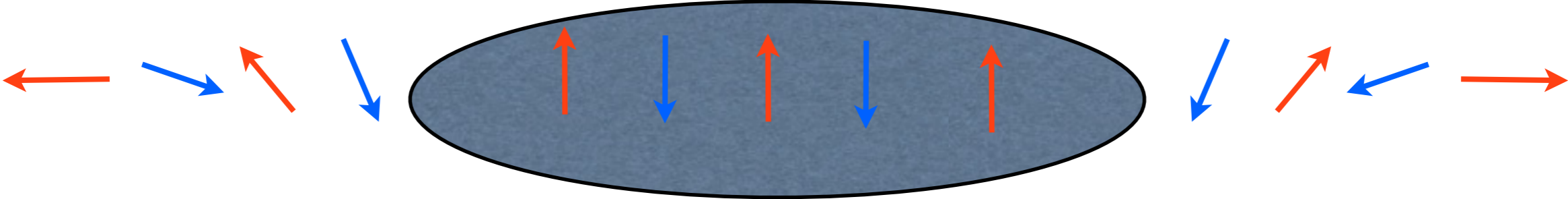
# 2D Heisenberg Antiferromagnets at Finite Temperature

Hohenberg-Mermin-Wagner  
Theorem (1966)



No Long-Range Order  
at Finite Temperatures

$$\xi \sim a e^{\frac{2\pi JS^2}{kT}}$$



Frustration



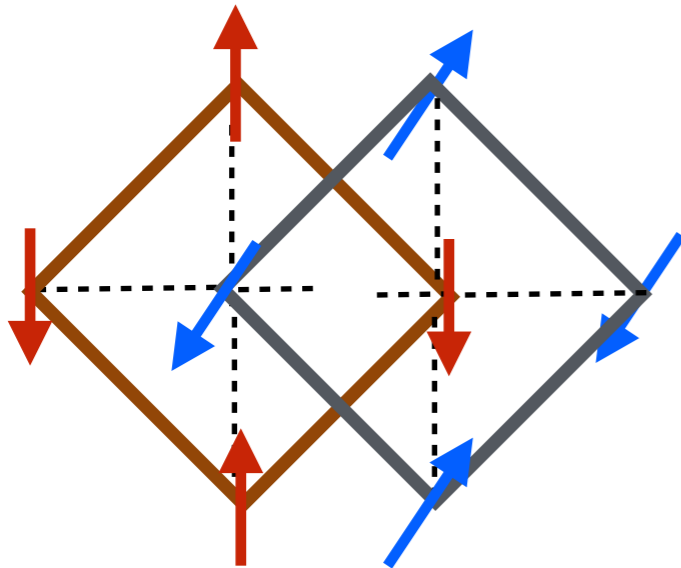
Discrete Symmetry-Breaking  
at Finite Temperatures

# Thermal Fluctuation-Selected Order: A Simple Example

$$H = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i \cdot S_j$$

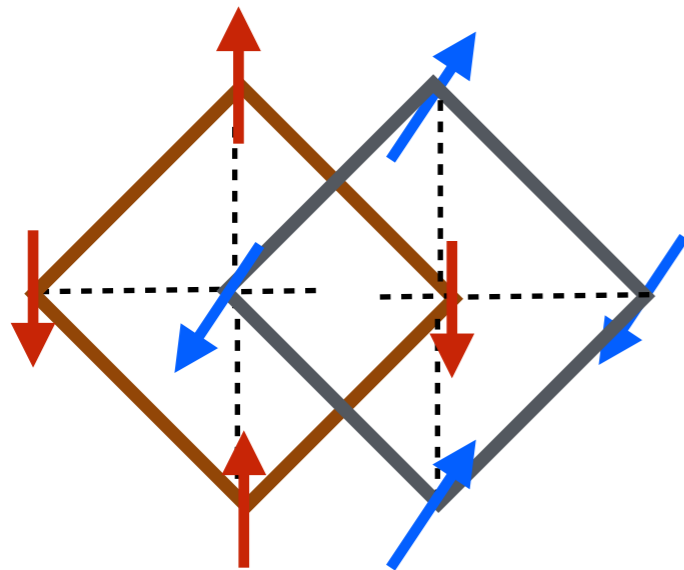
$$(J_2 \gg J_1)$$

Sublattices classically decoupled

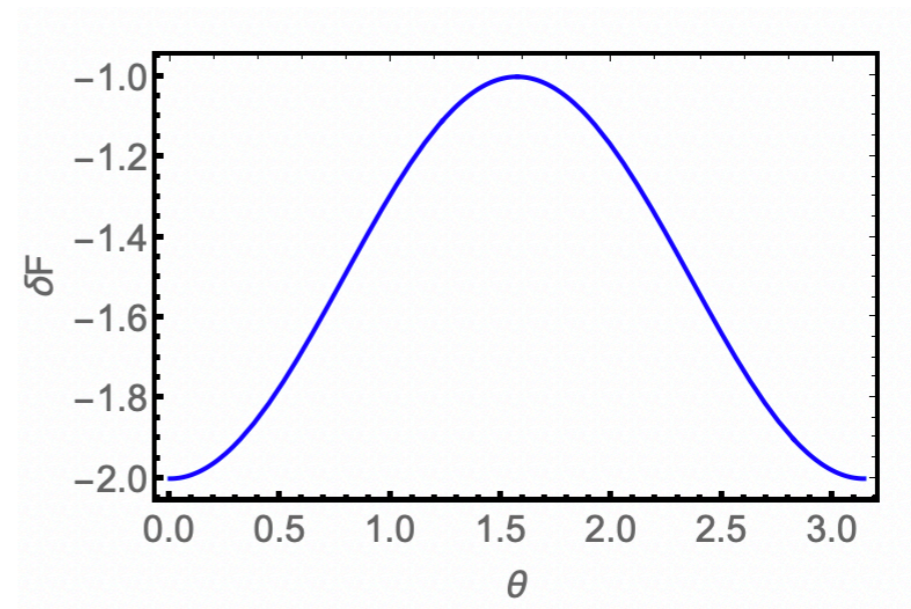


# Thermally Fluctuation-Selected Order: A Simple Example

$$H = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i \cdot S_j \quad (J_2 \gg J_1)$$



Fluctuation Free Energy

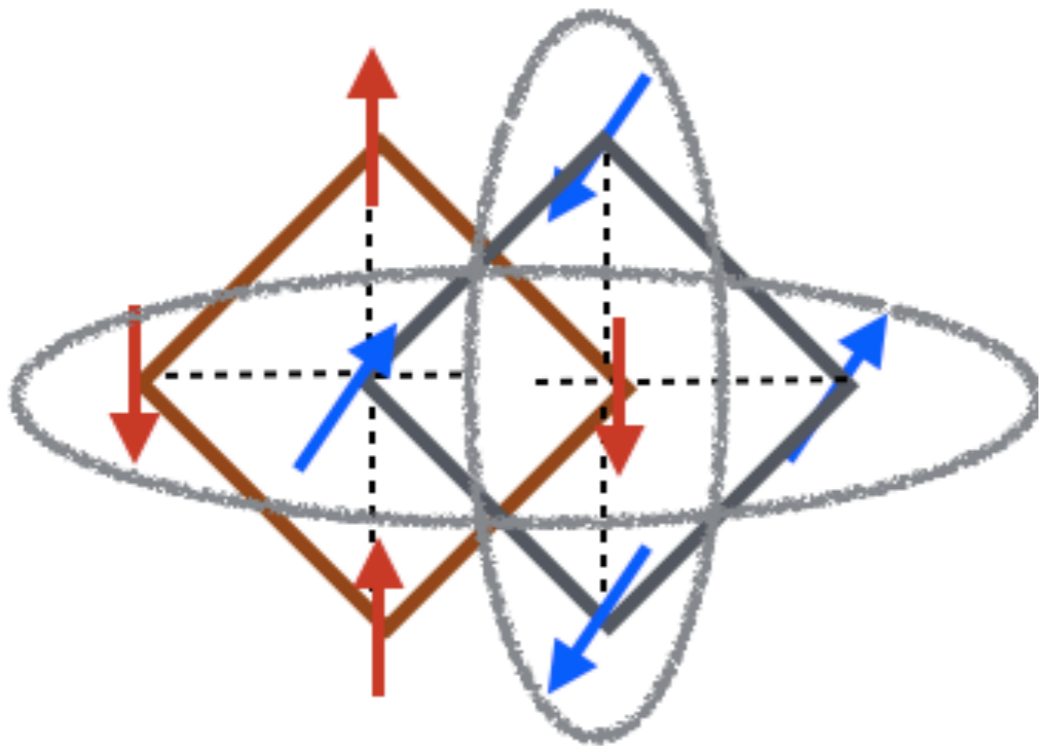


Collinear States Selected

# Thermally Fluctuation-Selected Order: A Simple Example

$$H = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i \cdot S_j \quad (J_2 \gg J_1)$$

“Order from Disorder”  
(Villain 1977, Shender 1982, Henley 1989)



Discrete  $Z_2$  Relative Degree of Freedom

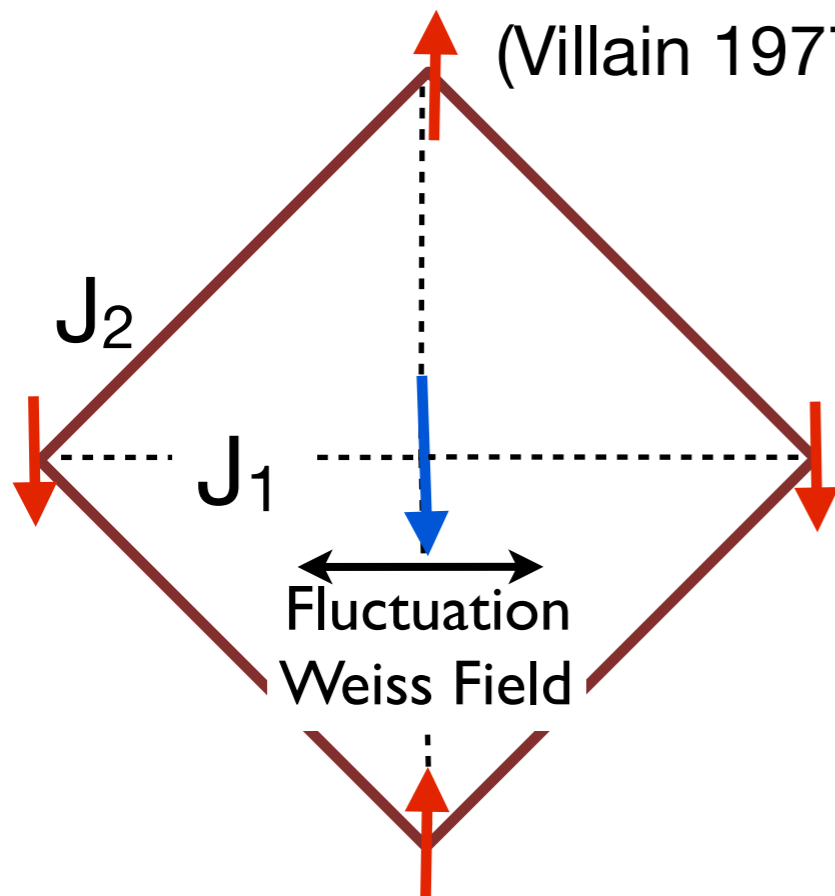
$$\vec{M}_1 \cdot \vec{M}_2 = \sigma = \pm 1$$

# Thermally Fluctuation-Selected Order: A Simple Example

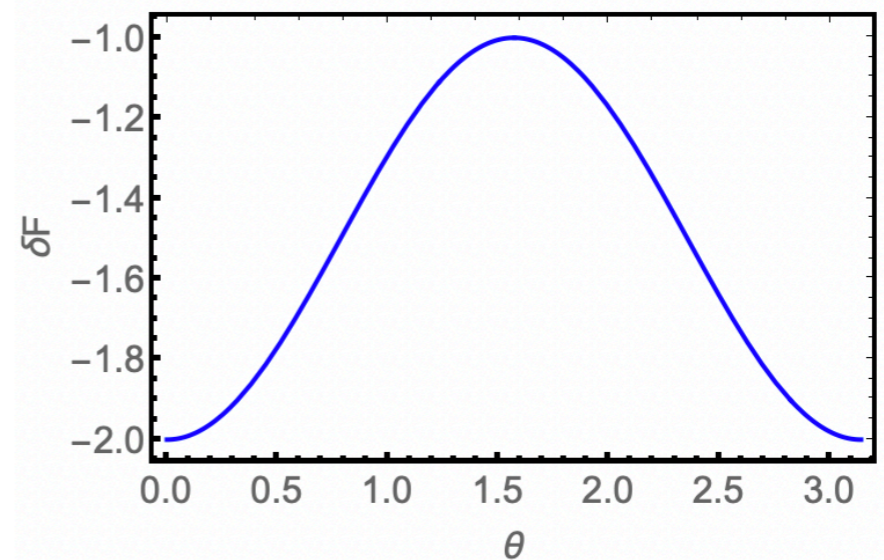
$$H = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i \cdot S_j \quad (J_2 \gg J_1)$$

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Fluctuation Free Energy

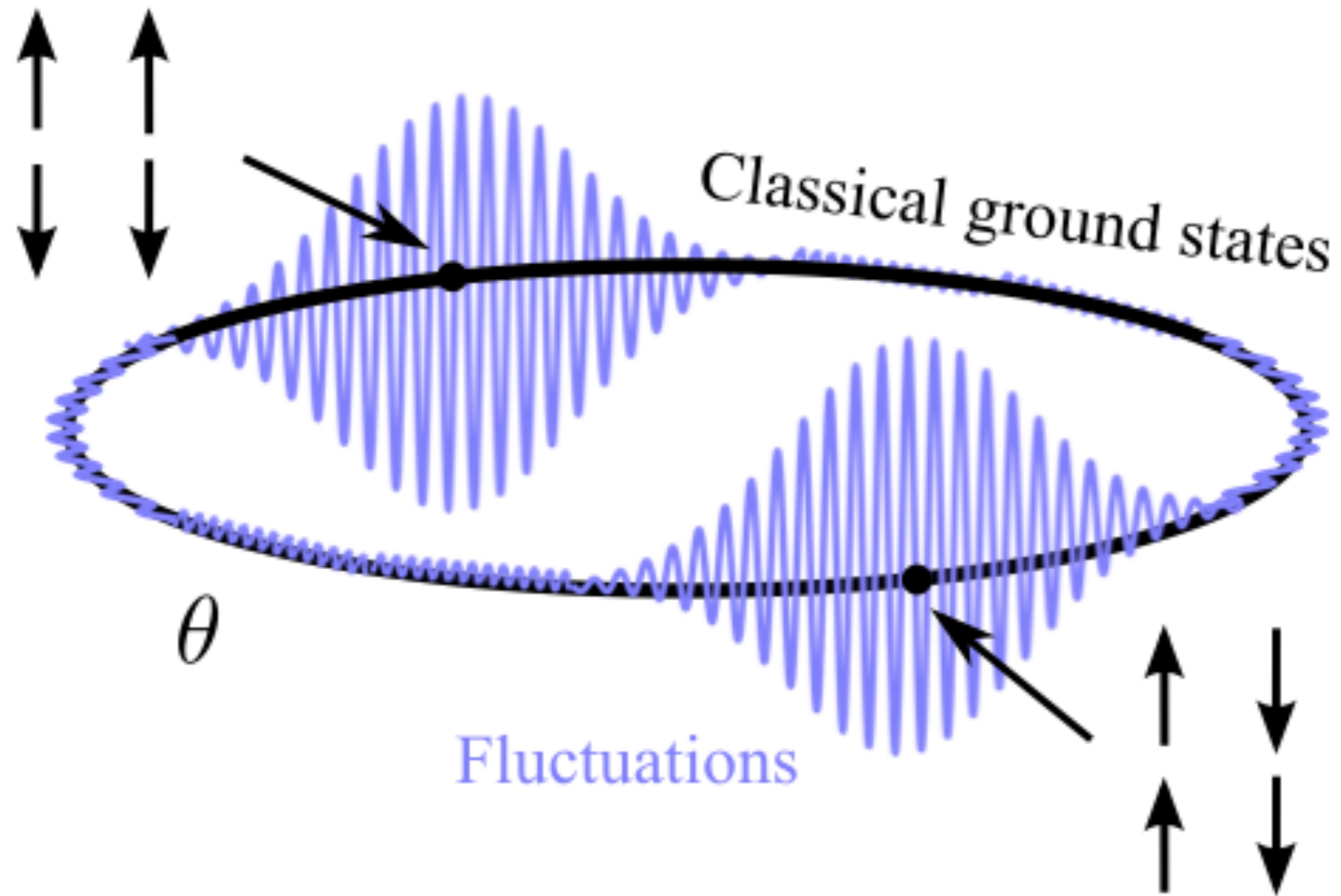


Spins like to align the **fluctuation** Weiss field to their “easy plane” (Henley 1989),

Collinear States Selected



# Order From Disorder



Fluctuations Select "Soft" States from Degenerate Ground-State Manifold

(courtesy, R. Flint)

(Villain 80, Shender 82, Henley 89)



# First Theory-Experiment Agreement on "Order from Disorder"

EUROPHYSICS LETTERS

1 September 1988

*Europhys. Lett.*, 7 (1), pp. 83-86 (1988)

## Quantum Exchange Magnon Gap in an Antiferromagnet with Dynamically Interacting Spin Subsystems.

A. G. GUKASOV (\*), TH. BRÜCKEL (\*\*), B. DORNER (\*\*), V. P. PLAKHTY (\*)  
W. PRANDL (\*\*\*) , E. F. SHENDER (\*) and O. P. SMIRNOV (\*)

(\*) *Leningrad Nuclear Physics Institute, Gatchina, Leningrad 188350, USSR*

(\*\*) *Institut Laue-Langevin, 156X, 38042 Grenoble Cedex, France*

(\*\*\*) *Institut für Kristallographie der Universität Tübingen  
Charlottenstr. 33, D-7400 Tübingen, BRD*

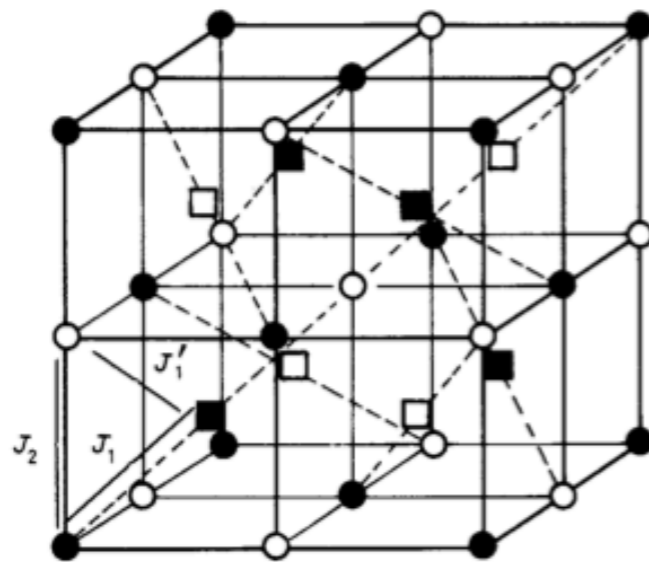
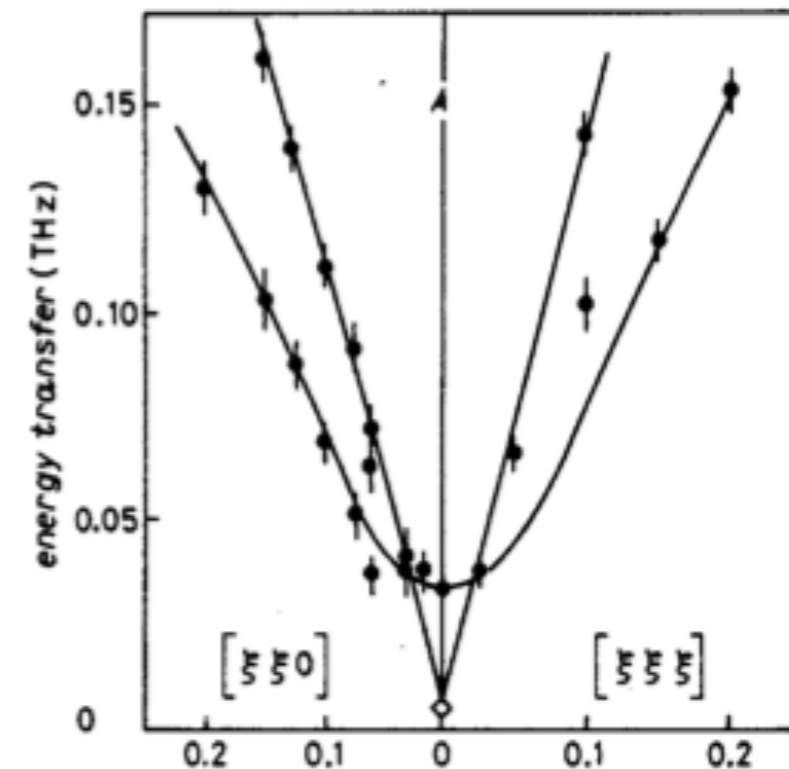


Fig. 1. - The magnetic structure of  $\text{Fe}_2\text{Ca}_3(\text{GeO}_4)_3$ . Open symbols indicate spin down, full symbols spin up. The two subsystems mentioned in the text are distinguished by squares and circles, respectively. Dashed lines indicate the threefold axis.



## Quantum Order by Disorder and Accidental Soft Mode in $\text{Er}_2\text{Ti}_2\text{O}_7$

M. E. Zhitomirsky,<sup>1</sup> M. V. Gvozdikova,<sup>1</sup> P. C. W. Holdsworth,<sup>2</sup> and R. Moessner<sup>3</sup>

<sup>1</sup>*Service de Physique Statistique, Magnétisme et Supraconductivité, UMR-E9001 CEA-INAC/UJF, 17 rue des Martyrs, 38054 Grenoble Cedex 9, France*

<sup>2</sup>*Laboratoire de Physique, École Normale Supérieure de Lyon, CNRS 69364 Lyon Cedex 07, France*

<sup>3</sup>*Max-Planck-Institut für Physik komplexer Systeme, 01187 Dresden, Germany*

(Received 2 April 2012; published 16 August 2012)

## Order by Quantum Disorder in $\text{Er}_2\text{Ti}_2\text{O}_7$

Lucile Savary,<sup>1</sup> Kate A. Ross,<sup>2</sup> Bruce D. Gaulin,<sup>2,3,4</sup> Jacob P. C. Ruff,<sup>2,5</sup> and Leon Balents<sup>6</sup>

<sup>1</sup>*Department of Physics, University of California, Santa Barbara, California 93106-9530, USA*

<sup>2</sup>*Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada*

<sup>3</sup>*Canadian Institute for Advanced Research, Toronto, Ontario M5G 1Z8, Canada*

<sup>4</sup>*Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario L8S 4M1, Canada*

<sup>5</sup>*The Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439, USA*

<sup>6</sup>*Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030, USA*

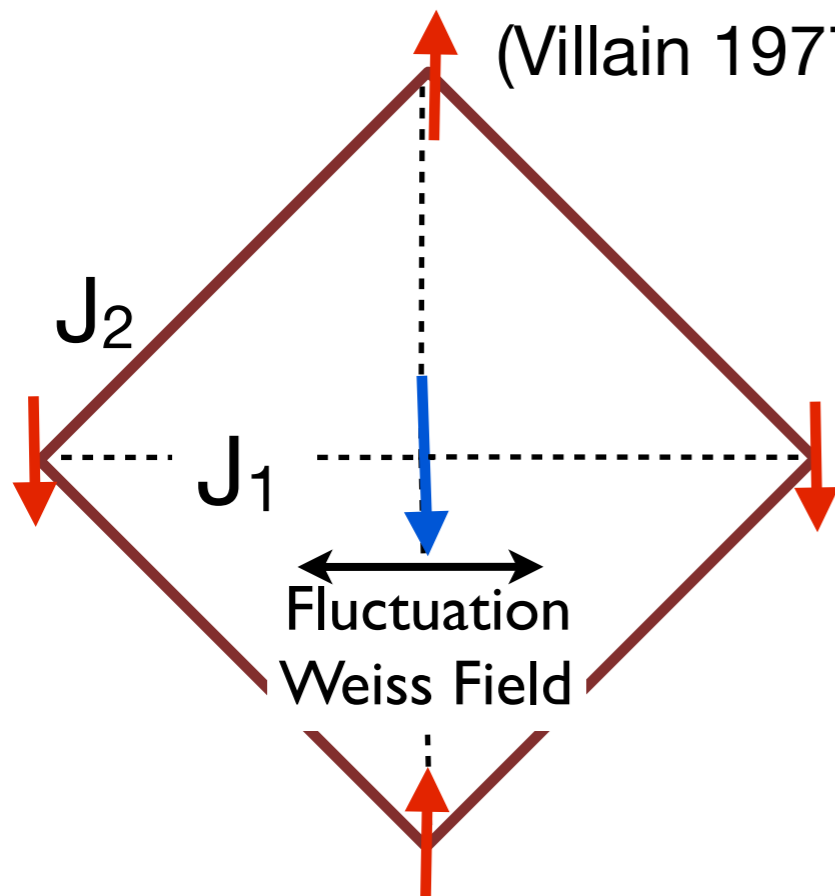
(Received 5 April 2012; published 15 October 2012)

# Thermally Fluctuation-Selected Order: A Simple Example

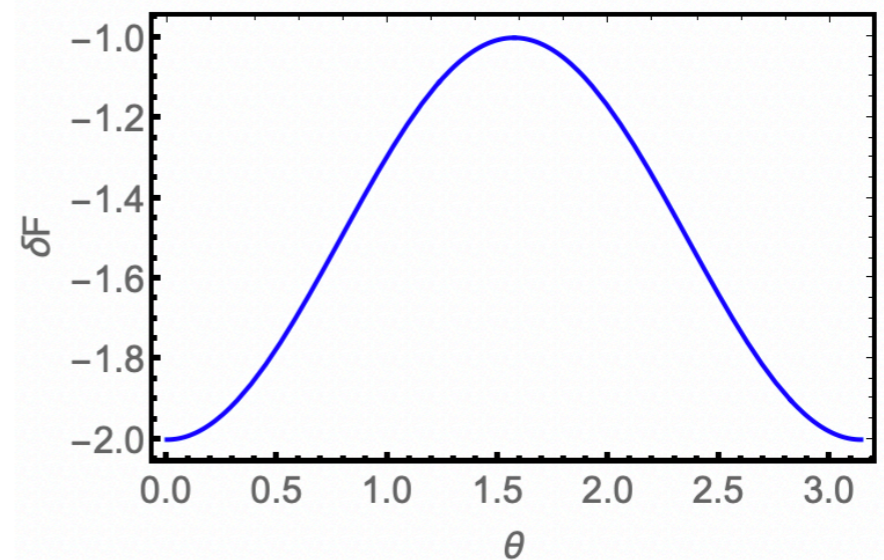
$$H = J_1 \sum_{\langle i,j \rangle} S_i \cdot S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i \cdot S_j \quad (J_2 \gg J_1)$$

“Order from Disorder”

(Villain 1977, Shender 1982, Henley 1989)



Fluctuation Free Energy



Collinear States Selected

Spins like to align the **fluctuation** Weiss field to their “easy plane” (Henley 1989),<sub>1,1</sub>

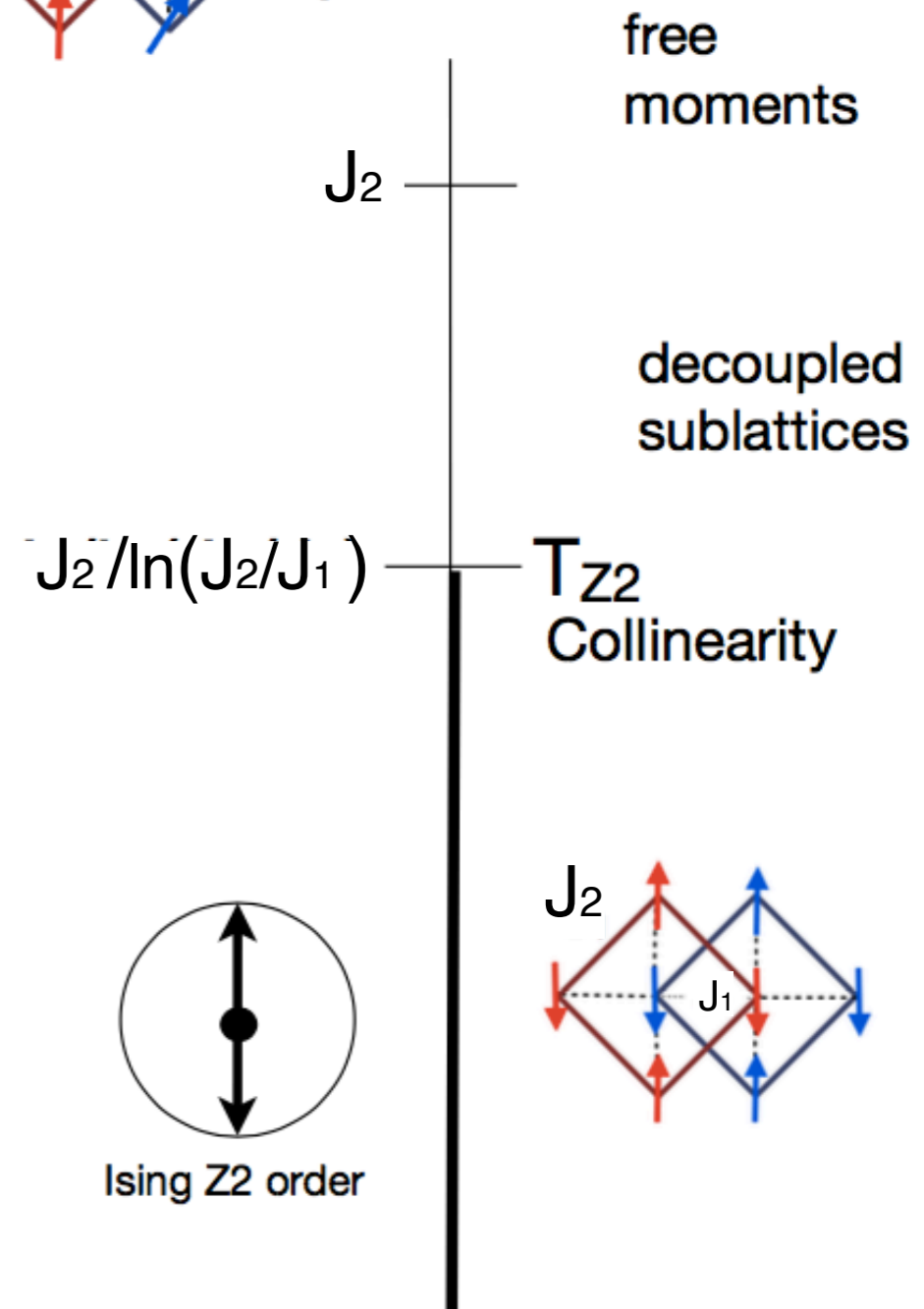
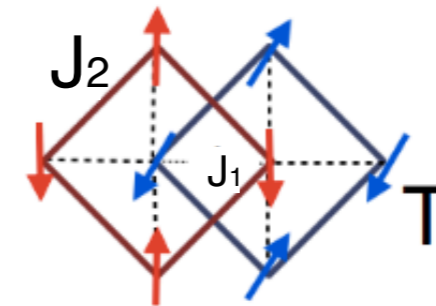
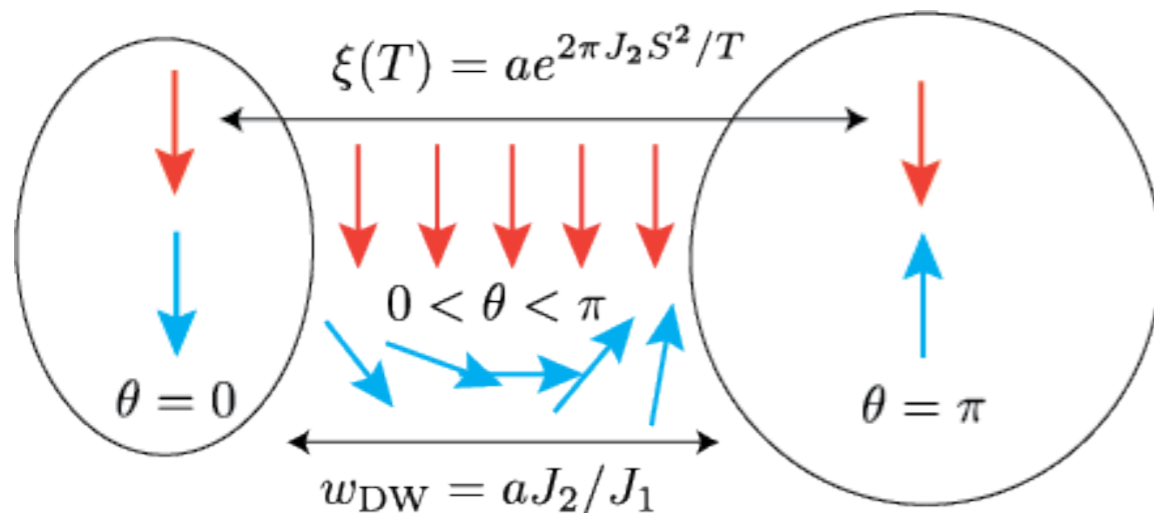
# Phase Diagram

$$S = \frac{1}{2} \int_x \sum_i K (\partial_\mu \mathbf{n}_i)^2 - \frac{\gamma}{a^2} \int_x (\mathbf{n}_1 \cdot \mathbf{n}_2)^2$$

Ising Transition Temperature  $T_c = J_2 / \ln(J_2/J_1)$

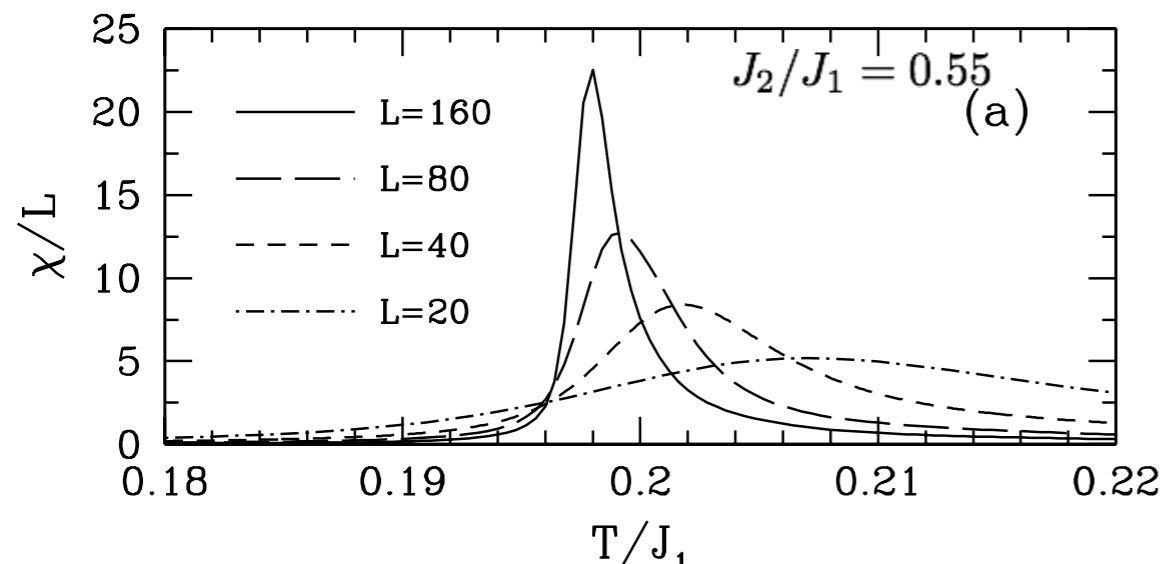
Intuitive Argument:  $Z_2$  domains defined if Heisenberg correlation length is longer than the domain wall thickness

$$\xi(T) = ae^{2\pi J_2 S^2 / T} > w_{\text{DW}} \propto aJ_2/J_1$$

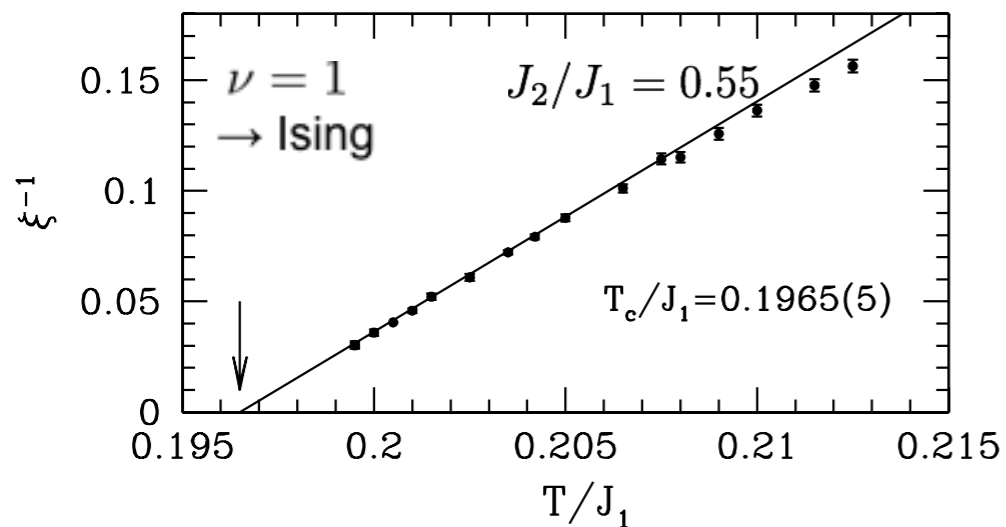
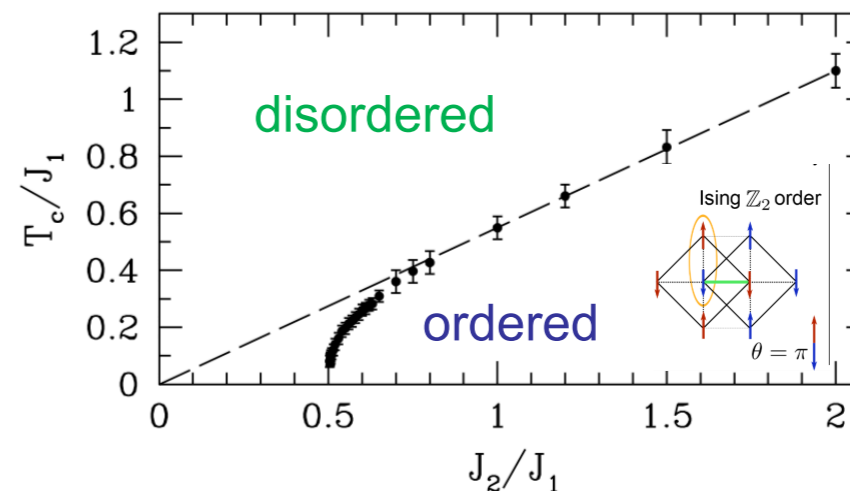


# Finite Temperature Ising Transition

Long Wavelength Analysis Confirmed by Classical Monte Carlo



MC phase diagram:



Weber et al (03)

Also confirmed for the S=1/2 case

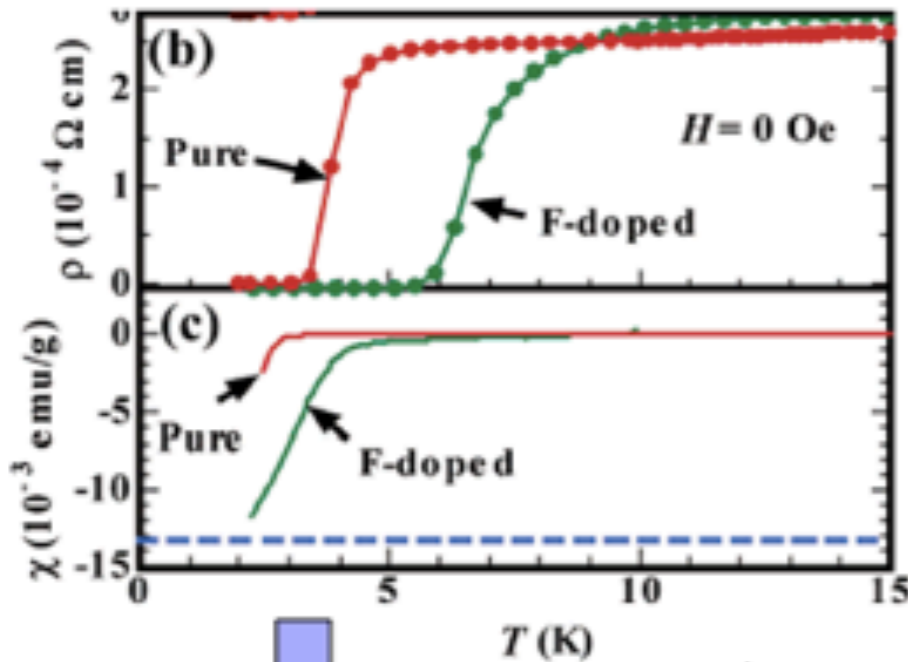
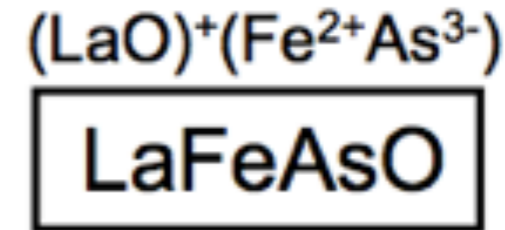
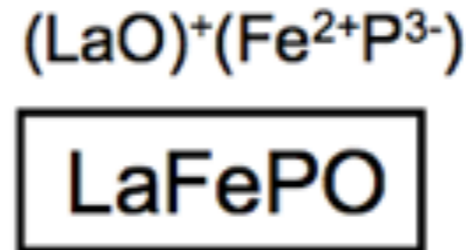
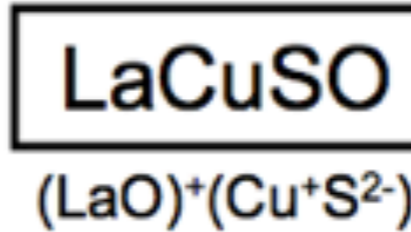
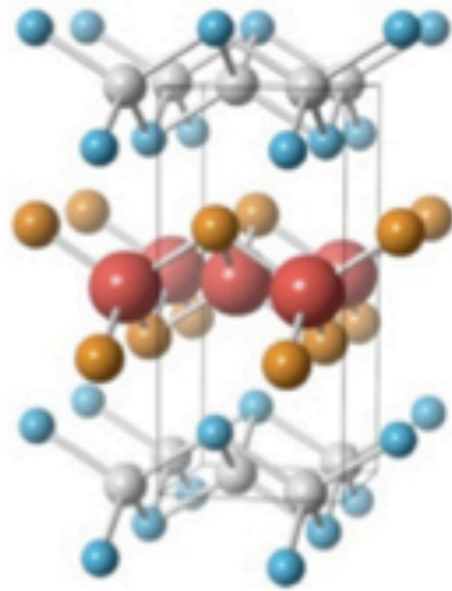
Any link with Experiment ??



# Discovery of New High- $T_c$ Superconductors



Hideo Hosono



Pure  
F substitution

F substitution



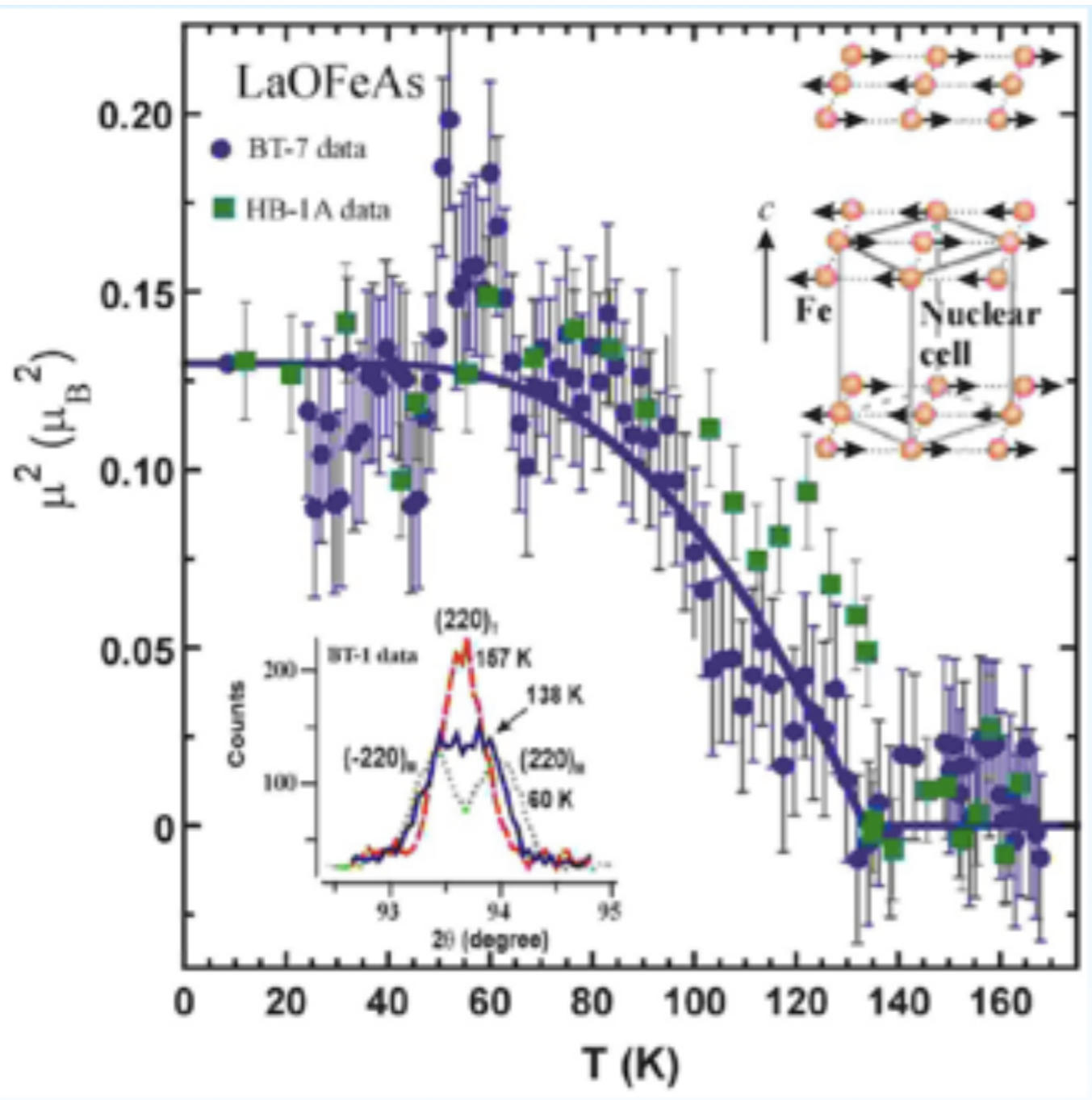
2007

2008

Rutgers  
Center for Materials Theory

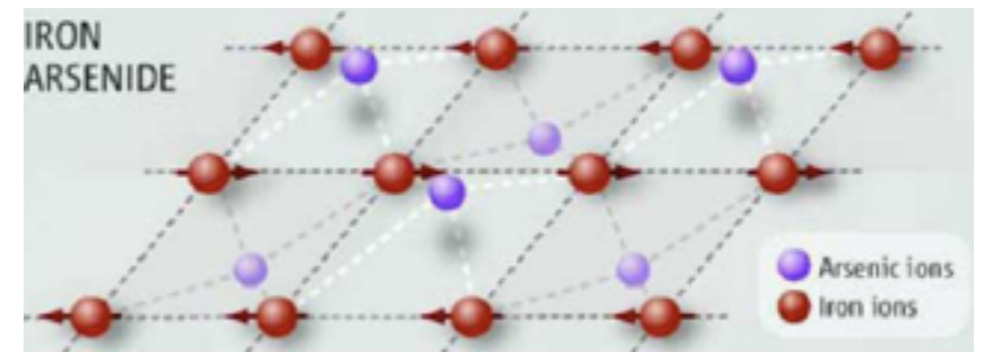
The Iron Age of Superconductivity has Begun !

# Spin Structure from Neutron Scattering



Cruz et al (08)

Familiar Spin Pattern !!



Frustration in the Pnictides  
(J1-J2 Model)

Yildirim (08)

Si and Abrahams (08)







P. Coleman (Rutgers)  
M. Continentino (CBPF)  
G. Lonzarich (Cambridge)

# Quantum Annealed Criticality

P. Chandra (Rutgers)

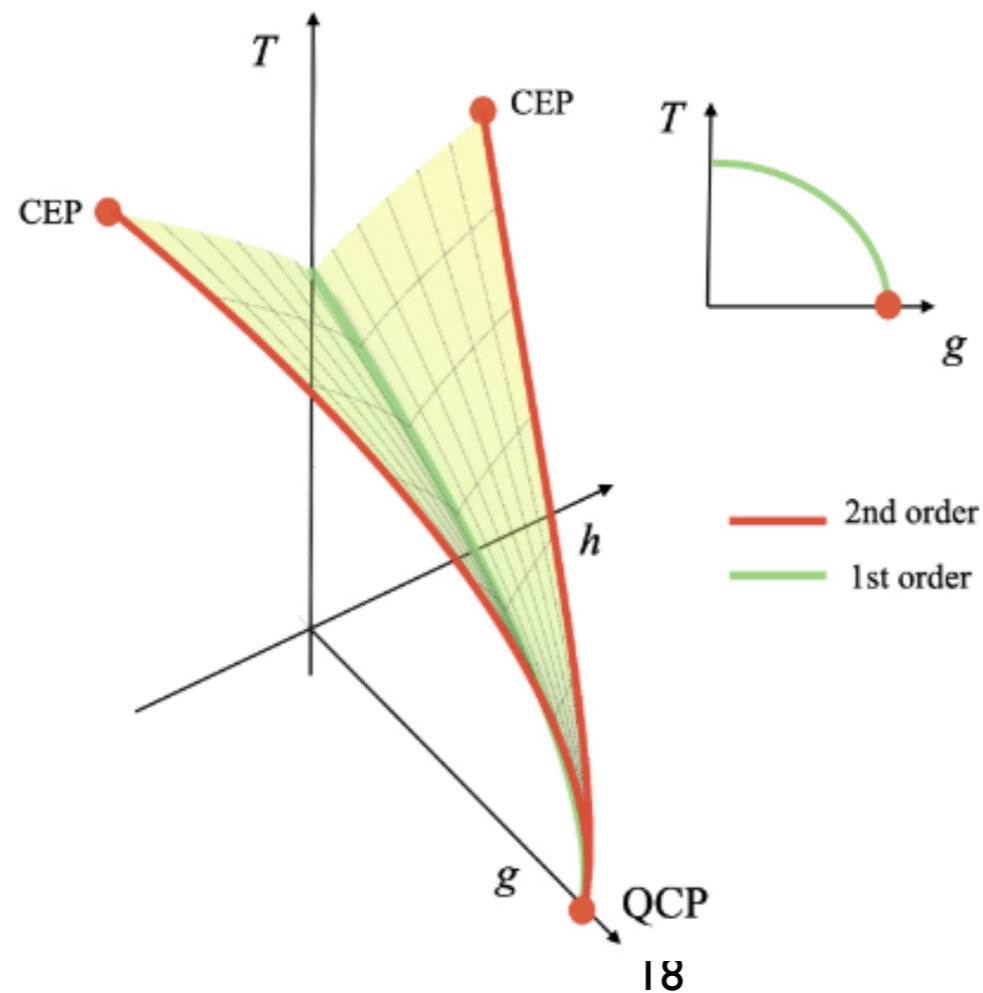
How can systems that have classical first-order transitions display quantum criticality ??

Phys. Rev. Research 2, 043440 (2020)



P. Coleman (Rutgers)  
M. Continentino (CBPF)  
G. Lonzarich (Cambridge)

# Quantum Annealed Criticality



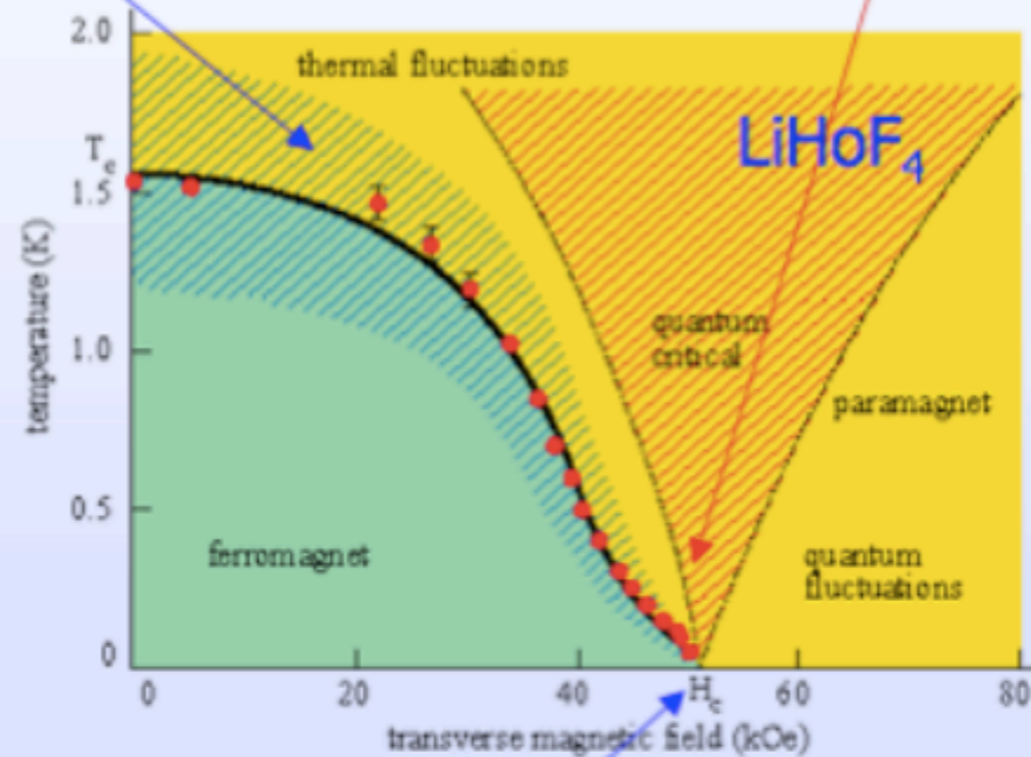
# Classical and Quantum Phase Transitions

classical phase transition

$$\xi \propto (T - T_c)^{-\nu_c} \quad (T \gg J)$$

quantum critical regime

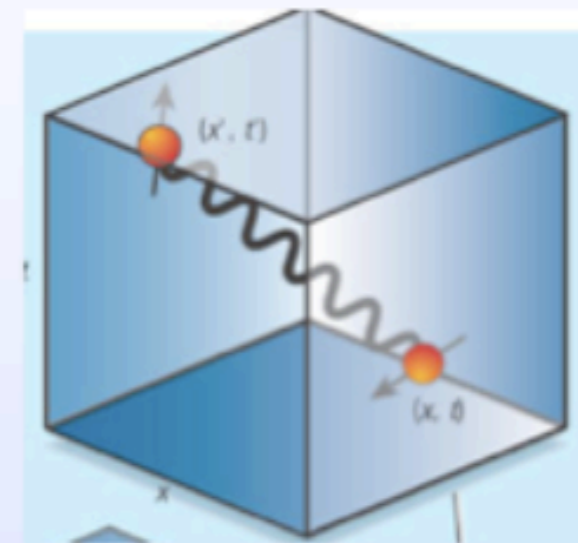
$$\xi(T) \propto T^{-\frac{1}{z}} \quad (T \ll J)$$



D. Bitko et al., Phys. Rev. Lett. 77, 940 (1996)

2<sup>nd</sup> order quantum phase transition

$$\xi(T = 0) \propto (g - g_c)^{-\nu}$$



$$d_{eff} = d + z$$

# Experimental Motivation: Ferroelectrics

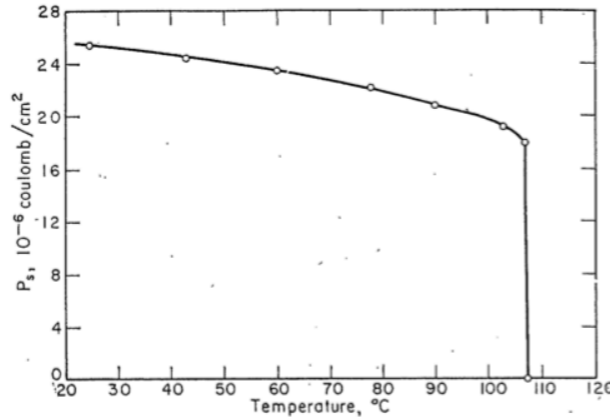
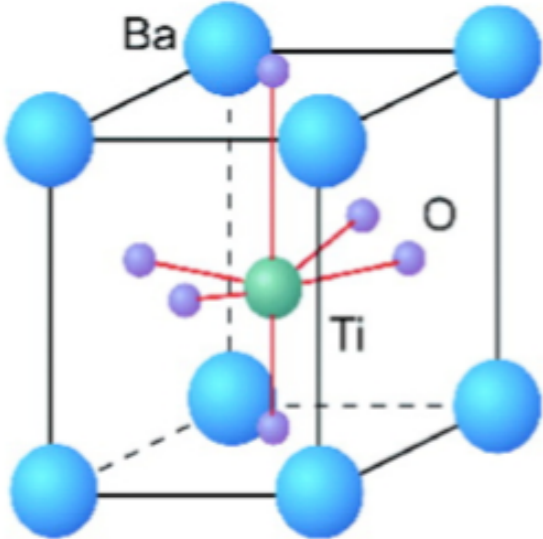
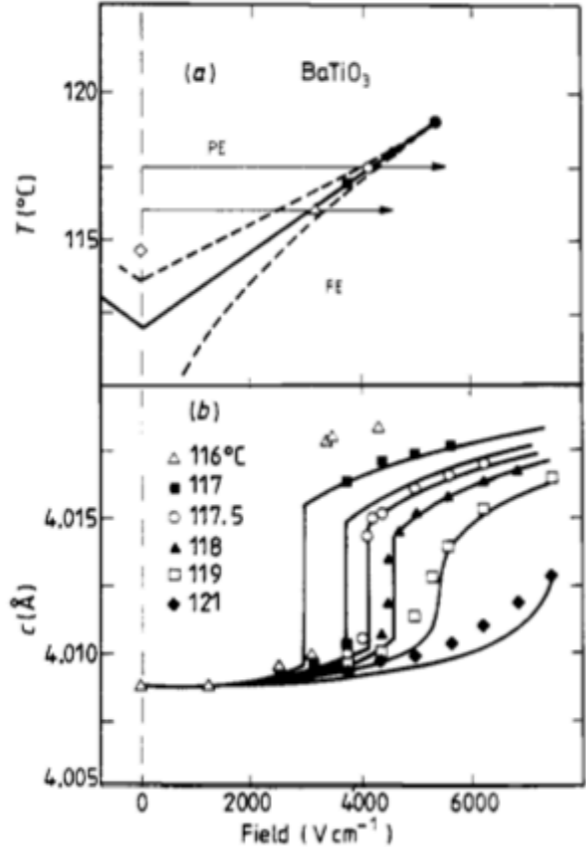


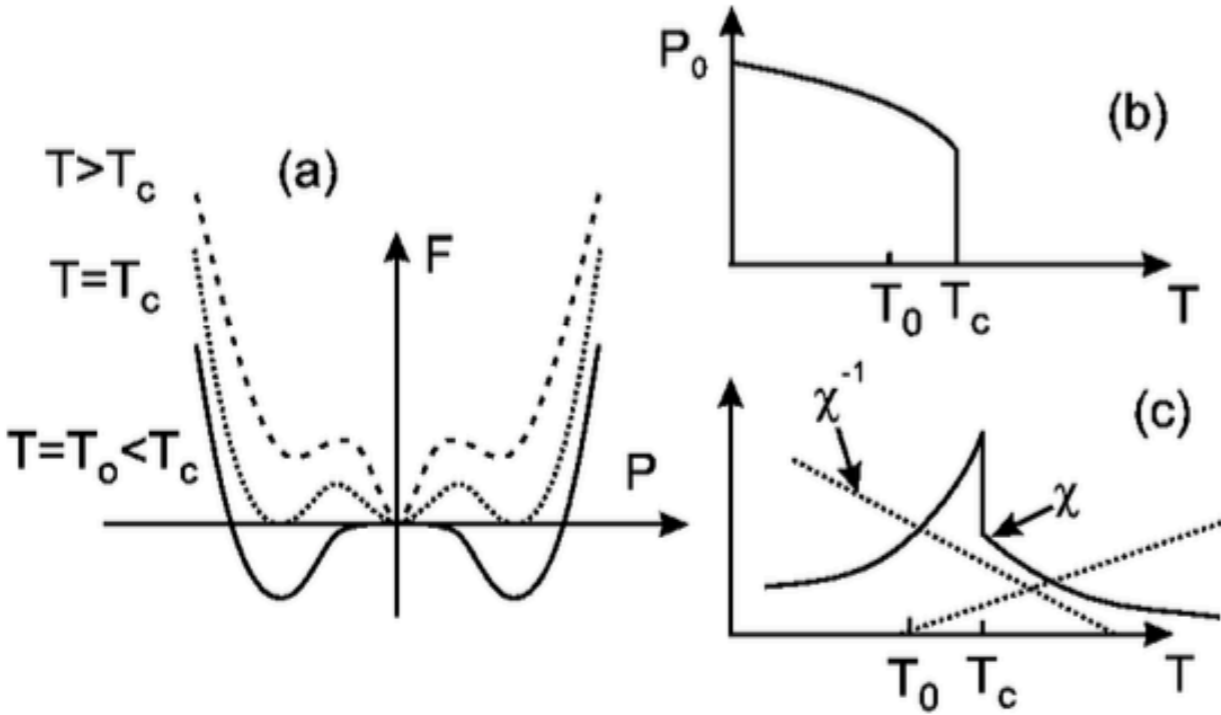
Fig. IV-7. Spontaneous polarization of tetragonal BaTiO<sub>3</sub> as a function of temperature (according to Merz (M 2)).

Jona and Shirane, FE Crystals (1962)



McWhan et al., J.Phys. C (1985)

Figure 1. (a) Phase diagram of BaTiO<sub>3</sub>, showing line of first-order transitions terminating at a critical point (full circle). (b) Lattice constant against electric field at different temperatures. Full and broken curves in 1(a) and 1(b) are calculated by minimising the free energy and they correspond to the equilibrium and spinodal boundaries.

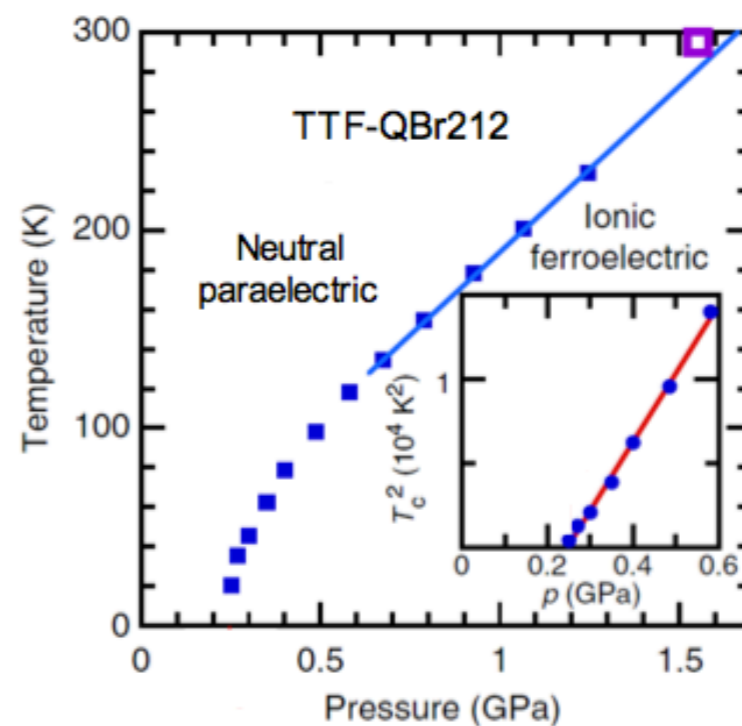
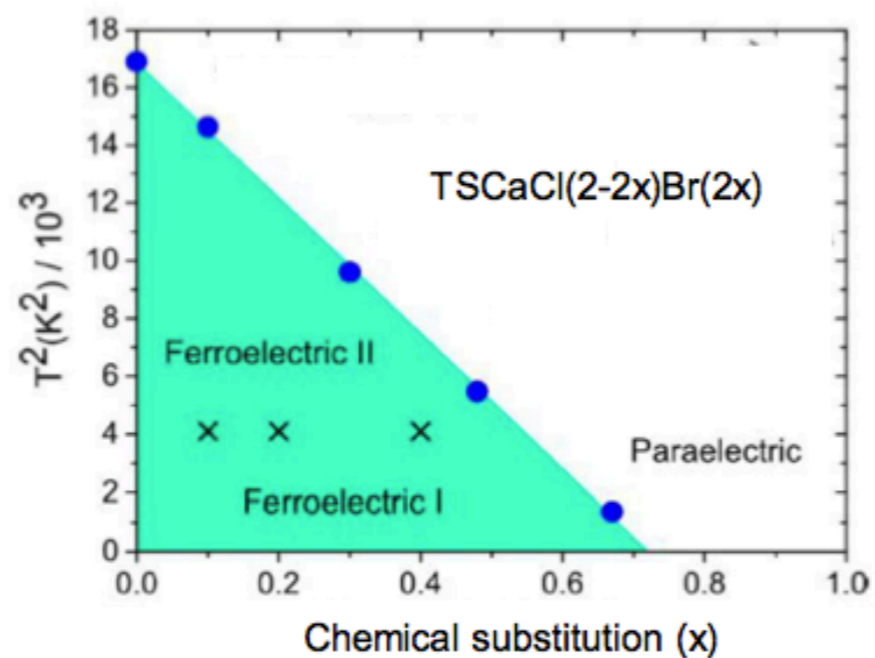
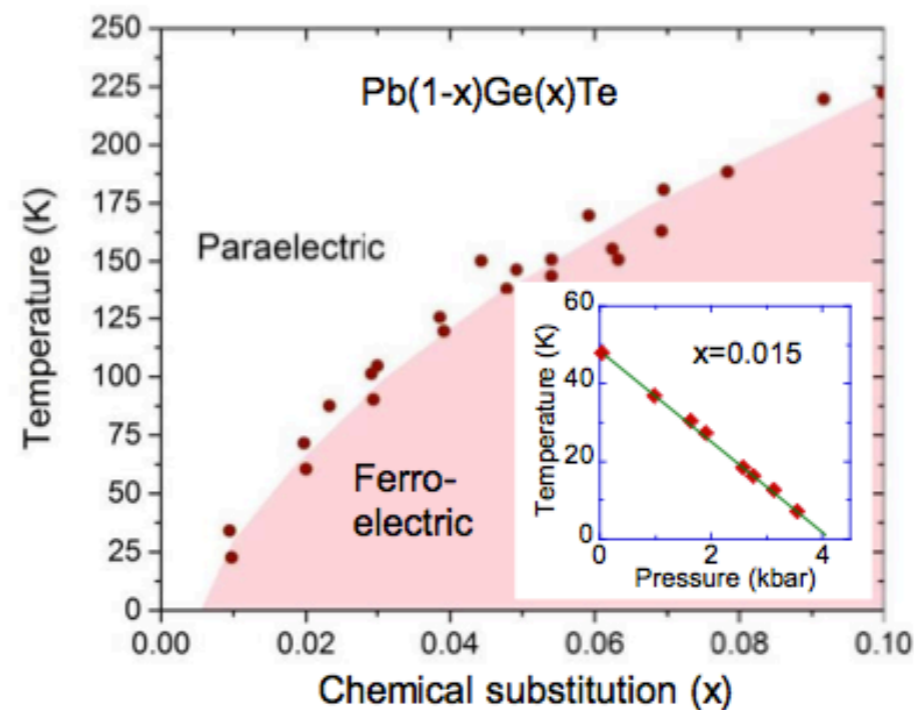
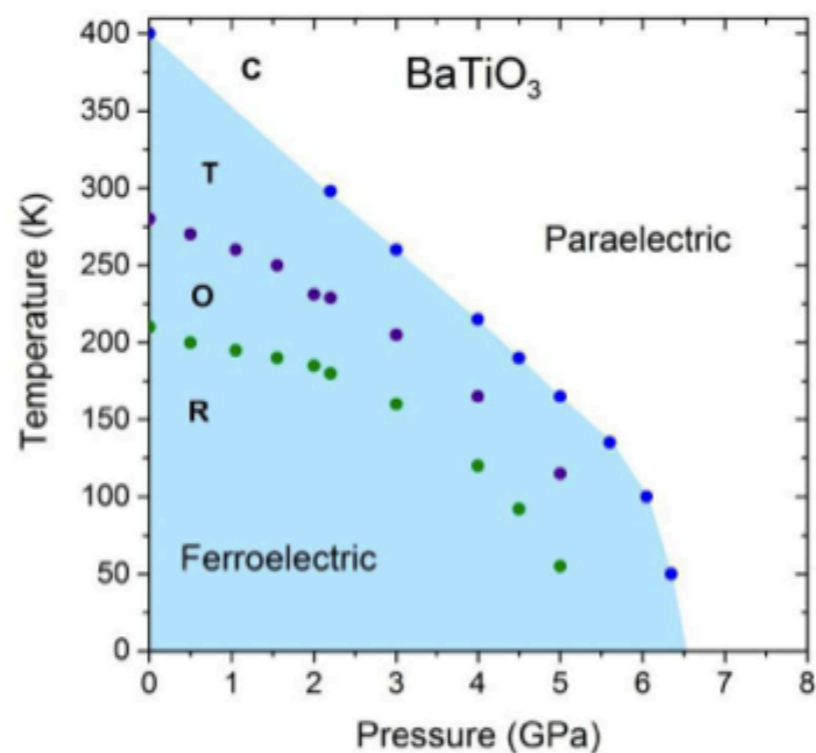


Classically First-Order !





# Quantum Criticality with Classical First-Order Transitions !





# (Classical) Larkin-Pikin Mechanism

(A. I. Larkin and S. Pikin, Sov. Phys. JETP 29, 891 (1969))



Interaction of strain with fluctuating critical order parameter

Diverging Specific Heat in a Clamped System



1st Order Transition in the Unclamped System

LP Criterion for 1st Order Transition

$$\kappa < \frac{\Delta C_V}{T_c} \left( \frac{dT_c}{d \ln V} \right)^2$$

$$\kappa^{-1} = K^{-1} - \left( K + \frac{4}{3} \mu \right)^{-1}$$

$$\kappa \sim K \frac{c_L^2}{c_T^2}$$

Shear Strain Crucial

Coupling of the uniform strain to the energy density



Macroscopic Instability of the Critical Point



Discontinuous Phase Transition

Generalization for the Quantum Case ???

## Generalization for the Quantum Case ???

Rewrite in terms of correlation functions (energy fluctuations)

Include dynamics and quantum tuning parameter

$$(\tilde{\kappa} = \kappa - \Delta\kappa) \quad \tilde{\kappa} < 0$$

1st Order Transition

### A Flavor for the Quantum Case

$$\lim_{T \rightarrow 0} \Delta\kappa \propto \int dq d\nu q^{d-1} \chi^2(q, i\nu)$$

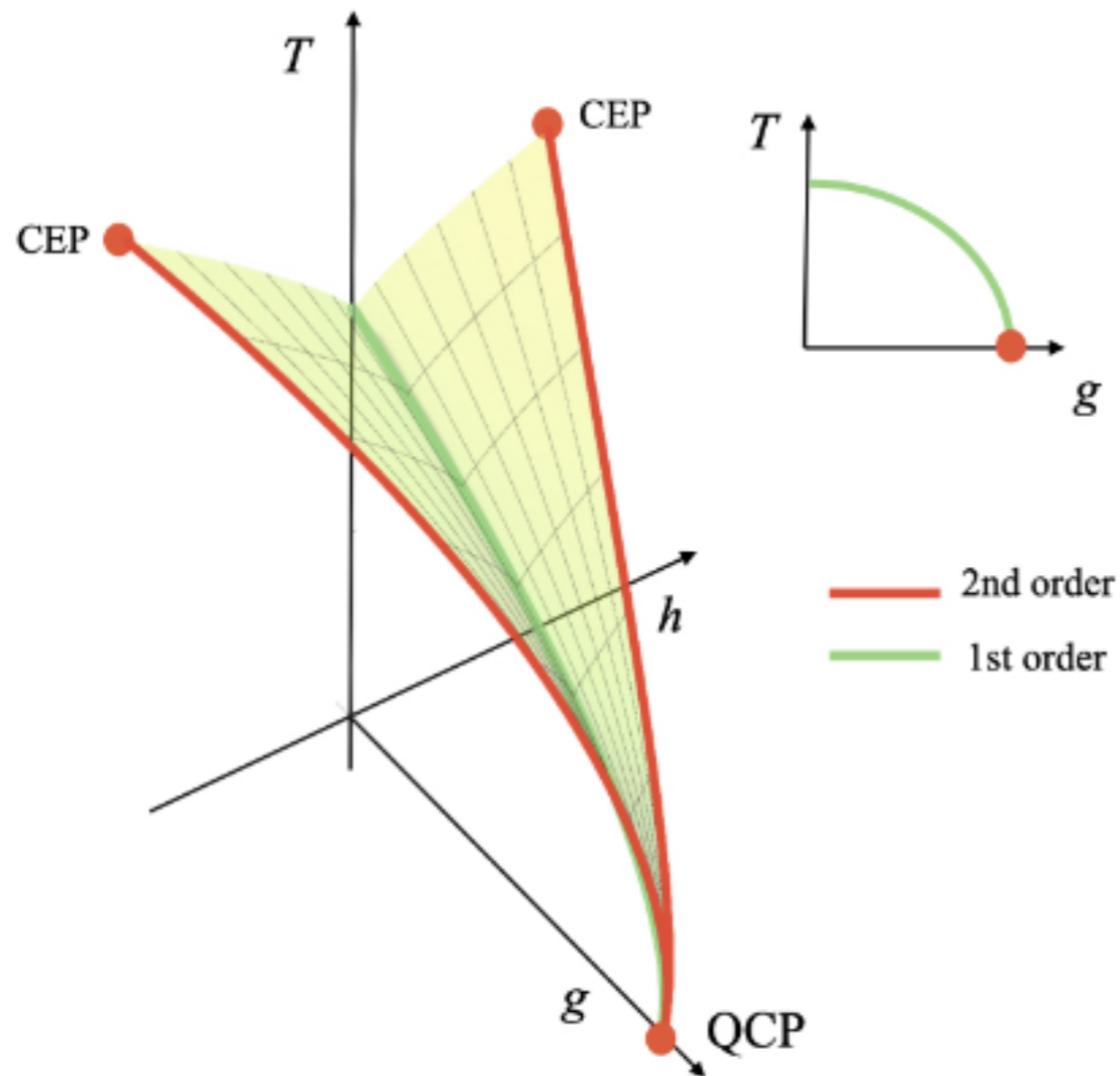
$$[\chi] = \left[ \frac{1}{q^2} \right]$$

$$\lim_{T \rightarrow 0} [\Delta\kappa] = \frac{[q^{d+z}]}{[q^4]}$$

$$[\nu] = [q^z]$$

Non-Singular for  $d + z > 4$  !!

# Generalized Larkin-Pikin Results



## Experimental Signatures

New Lattice-Sensitive

Settings for Exploration

of Exotic Quantum Phases

Elastic Anisotropy ??

Domain Dynamics??

Disorder ??

Metallic Systems ??

