2/4 More Frustrated Spin Systems

Spin Models = "Economy" Strongly Correlated Systems

AFM + frustration $\rightarrow$ Degenerate gd. state

$\downarrow$

competing interactions

Large Ground State Degeneracy (extensive)

- spins highly correlated but fluctuating strongly
- classical and/or quantum fluctuations

remarkable collective phenomena

emergent gauge fields

fractional particle excitations

Frustration is a short wavelength phenomenon associated with the geometry of the lattice.

Local properties (constraint geometry) $\Longrightarrow$ Long wavelength behavior

Effect of Temperature on degenerate ground state manifold?

<u>Order by Disorder</u>

Degenerate ground state is very sensitive to ground-state fluctuations

Thermal (and quantum) fluctuations select an ordered phase from the ground-state manifold

$$F = E - TS$$

$S \downarrow$ maximize entropy.

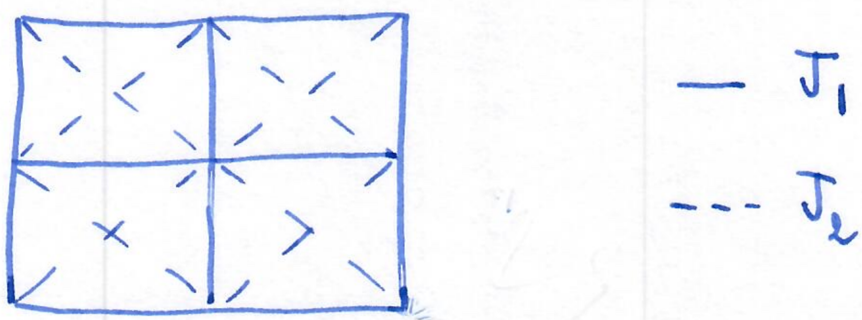
* Another possibility

finite T
↓
spin freezing!
(system gets stuck!)

spin glasses
↓
neural networks
↓
machine learning

• A Simple Example of Order by Disorder

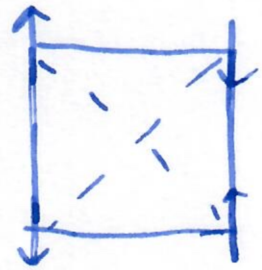
$$H = J_1 \sum_{nn} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{nnn} \vec{S}_i \cdot \vec{S}_j$$



Classically

• $J_1 >> J_2$

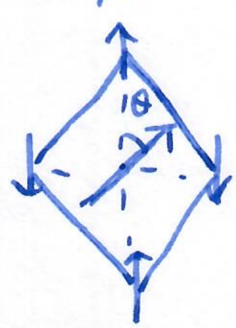
$$E = S^2 \left\{ -2J_1 + 2J_2 \right\}$$



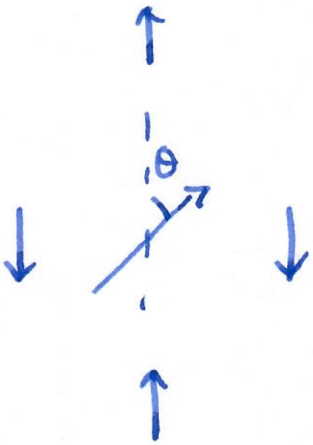
• $J_2 >> J_1$

$$E = S^2 \left\{ -2J_2 + 2\cancel{J_1} \cos\theta - 2\cancel{J_1} \cos\theta \right\}$$

classically the two sublattices are decoupled.



Spin Wave Theory



$O(1/zS)$

$$\omega(q,\theta) = (4J_2 S)\, f(q_x, q_y, \theta)$$

$$\delta F(\theta) = F(\theta) - F$$

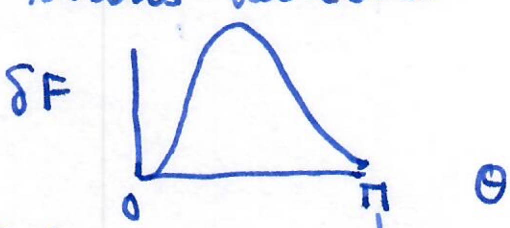
$$\downarrow$$

$$F(\theta) = T \sum \ln \frac{\omega(q,\theta)}{T}$$

$$\downarrow \quad \frac{J_2}{J_1} \gg 1$$

$$\boxed{\delta F(\theta) = -E(T)\left\{1 + \alpha \cos^2\theta\right\}}$$

2 states selected



Fake zero mode associated w/ "accidental soft" mode

Discrete degree of freedom

$\vec{M}_1 \cdot \vec{M}_2 = \sigma = \pm 1.$

<u>Favors</u> $\cos\theta = \pm 1$

↑ ↑	↑ ↓
↓ ↓	↑ ↓
↑ ↑	↑ ↓

FM $\omega \sim q^2$

AFM $\omega \sim q$

Fluctuation - Selected Order (2d)

$$\xi \sim e^{J_2 S^2 / T}$$ finite at $T \neq 0$

Despite finite correlation length
(no long-range Heisenberg AFM)

$\Downarrow$

emergent Z_2 long-range order

$\Downarrow$

emergent Z_2 phase transition in a disordered 2D Heisenberg system!

- $\mathbb{Z}_2$ order parameter

$$\sigma = \frac{(\vec{S}_1 - \vec{S}_3)\cdot(\vec{S}_2 \cdot \vec{S}_4)}{|S|^2}$$

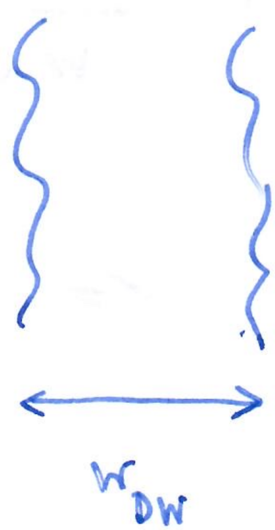
order in relative spins

(ξ finite at $T \neq 0$)

$\boxed{\text{Example of composite order}}$

- Phase Diagram?

$\mathbb{Z}_2$ domains OK if domain walls are defined



w_{DW}

$\xi \sim w_{DW}$ estimate of stability

W_{DW} ?

$$S \sim \int_x J_2 \sum_i (\partial_\mu n_i)^2 - \frac{\gamma}{a^2} \int_x (n_1 \cdot n_2)^2$$

$$\Delta E = - \underset{\substack{\uparrow \\ \sim \frac{1}{J_2}}}{\frac{\chi}{2}} \underset{\substack{\uparrow \\ J_1^2}}{\langle B_\perp^2 \rangle}.$$

$$\frac{E_{DW}}{a} \sim \left\{ \frac{J_2}{W} + \left(\frac{J_1^2}{J_2} \right) \frac{W}{a^2} \right\}.$$

$$\frac{dE_{DW}}{dW} = 0 \Rightarrow -\frac{J_2}{W^2} + \frac{J_1^2}{J_2 a^2} = 0$$

$$\Downarrow$$

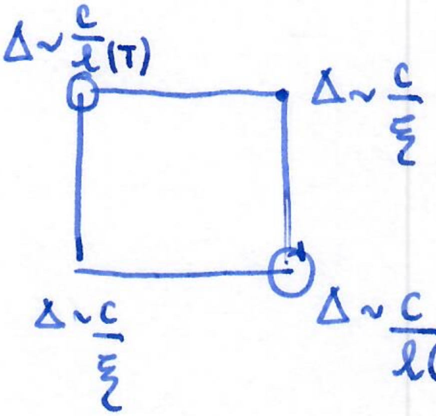
$$W \approx \left(\frac{J_2}{J_1} \right) a$$

Condition for stability of Z_2 order

$$e^{2\pi J_2 S^2 / T_c} \sim \frac{J_2}{J_1}$$

$$\Downarrow$$

$$T_c = \frac{2\pi J_2 S^2}{\ln\left(J_2/J_1\right)}$$



$\checkmark$ $J_1 = 0$

$T_c \to 0$

- Link to experiment $\to$ pnictides

  Nematic and structural (orthorhombic) transitions coincide

  (and both appear to be driven by magnetic fluctuations)

How to probe nematic fluctuations?

shear

$$H_{mag-el} = \lambda \varphi \varepsilon_s \qquad \varepsilon_s = \frac{\varepsilon_{aa} - \varepsilon_{bb}}{\sqrt{2}}$$

$$c_s^{-1} = c_{s0}^{-1} + \lambda^2 c_{s0}^{-2} \chi_{nem}.$$

Connection to Superconductivity ?!

Fluctuation - Selected Z_2 order

Emergent Composite Order

(Why It's So Hard to Identify a Classical Spin Liquid)

Order by Disorder

Thermal

$F = E - TS$

entropy weight ⇒ gd. state selection from each degenerate manifold

Fluctuations ⇒ Ordered State

$$S = S_\theta + S_D$$

S_θ ↓ continuous fluctuations of each spin (scales w/ N)

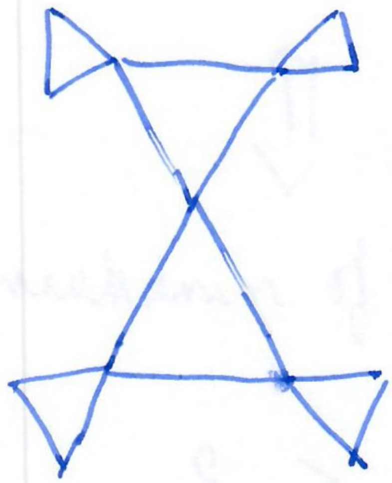
S_D ↓ configurational (discrete)

usually

$$S_\theta > S_D$$

↑ difficult to determine analytically.

Exact enumeration allowed in recursive structures (e.g. Husimi cactus)



S_D ~ "Delocalization Bandwidth" W

S_θ ~ RMS of random part of potential $\sqrt{\overline{\delta V_r^2}}$ inhibiting diffusion

Here using analogy w/ electronic propagation in disordered medium

Delocalization criterion

$$W > \sqrt{\overline{\delta V_r^2}}$$

$$\Updownarrow$$

Breakdown of order by disorder

$$S_D > S_O$$

S_D <u>must</u> scale w/ N

Then <u>prefactors</u> crucial

entropy (unlike electronic energy)
no time scale!

Usually
Fluctuations → Discrete Order
(frustrated spin system)

Short wave-length → breaks lattice Symmetry
fluctuations in frustrated system

↓

Long-Range Discrete Order