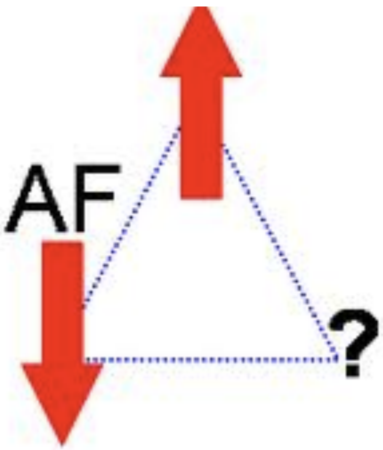
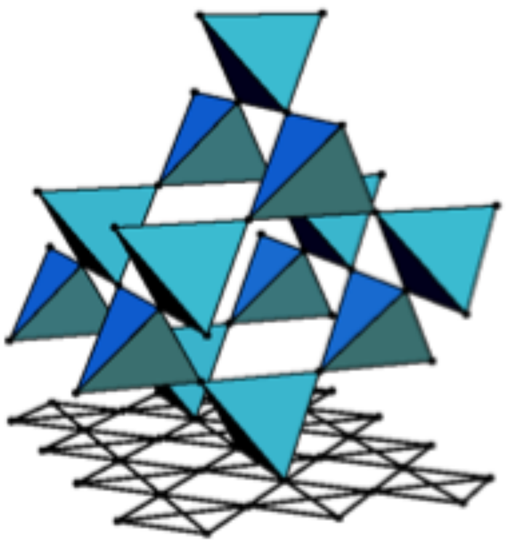
Introduction to Classical Frustrated Magnetism

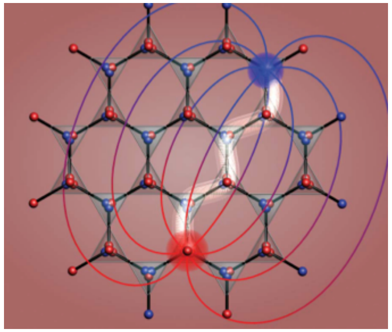


Magnetism: A Brief History

What is Frustration (for Spins) ??

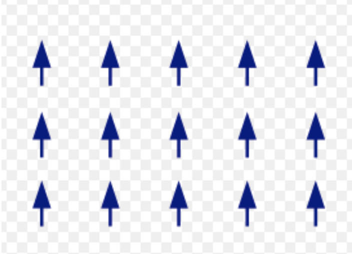


Spin Ice Basics



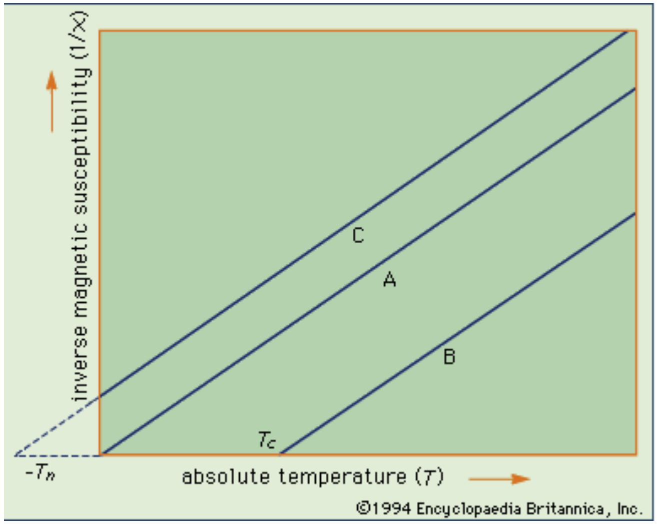
Magnetic Monopoles in Spin Ice

Ferromagnetism



Thermal Properties (Curie,Weiss)

$$\chi = \lim_{H \to 0} \frac{\mathcal{M}}{H} = \frac{C}{T}$$



Quantum Mechanics Necessary
for Magnetism!!
(Bohr-van Leeuwen Theorem)

Heisenberg Model

$$\mathcal{H} = -\sum_{i,j} \mathcal{J}_{ij} \vec{s}_i \cdot \vec{s}_j$$

MnO: An Experimentally Verified AFM!

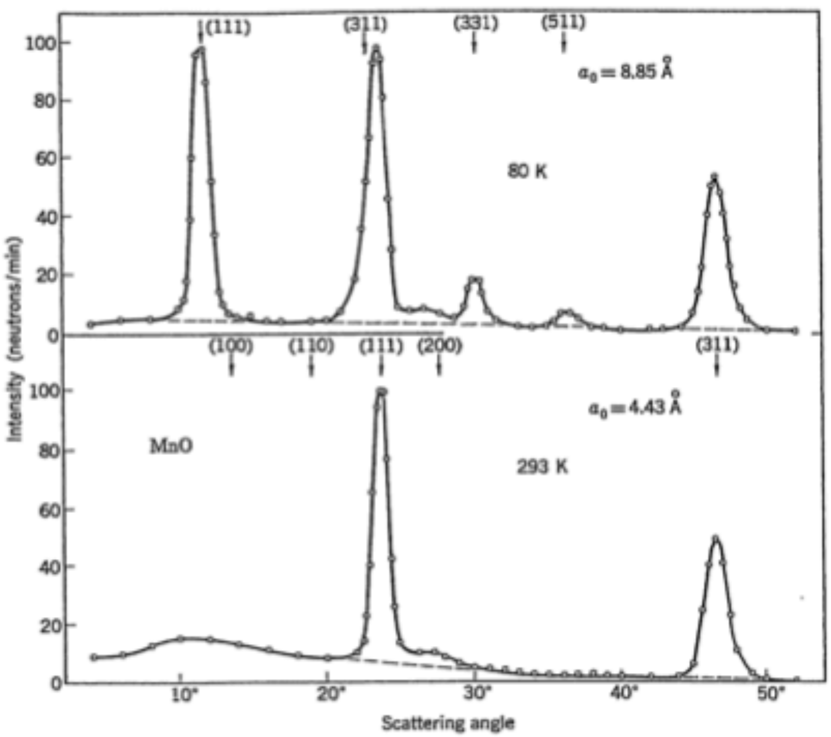


Figure 20 Neutron diffraction patterns for MnO below and above the spin-ordering temperature of 120 K, after C. G. Shull, W. A. Strauser, and E. O. Wollan. Phys. Rev. 83, 333 (1951). The reflection indices are based on an 8.85 Å cell at 80 K and on a 4.43 Å cell at 293 K. At the higher temperature the Mn^{2+} ions are still magnetic, but they are no longer ordered.

More neutron reflections at T = 80 K than at T = 240 K !

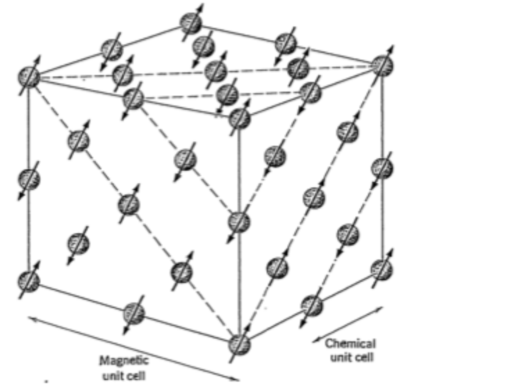
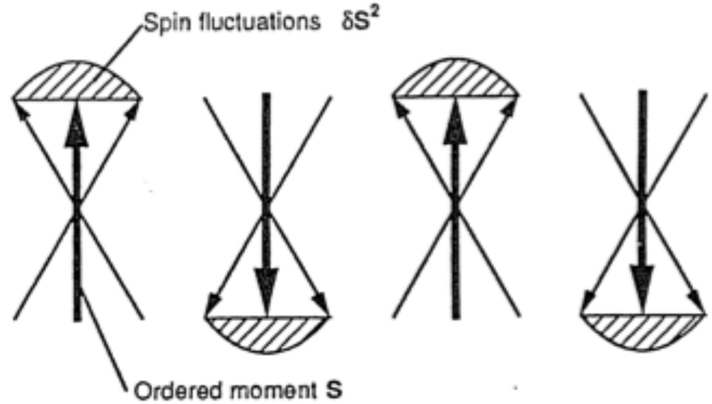


Figure 21 Ordered arrangements of spins of the Mn^{2+} ions in manganese oxide. MnO, as determined by neutron diffraction. The O^{2-} ions are not shown.

Shull, Strauser and Wollan

AFM = Semiclassical Spins + Zero-Point Fluctuations



"Melting" argument

$$\delta M \sim \int d^d q \left[n(\omega_q) + \frac{1}{2S} \right] \frac{1}{\omega_q}$$

$$< x^2 >_{CM} \sim \frac{T}{\omega^2}$$

$$< x^2 >_{QM} \sim \frac{\hbar}{2\omega}$$

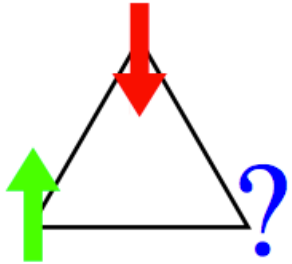
Neel good for d > 1 !!!

(Anderson, Kubo)

Frustration !!

Consider Ising spins $\sigma_i = \pm 1$ with antiferromagnetic $J > 0$:

$$\mathcal{H} = J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$



- Not all terms in $\mathcal{H}$ can simultaneously be minimised

- ground-state condition: $\uparrow\uparrow\downarrow$ or $\uparrow\downarrow\downarrow$ for each triangle

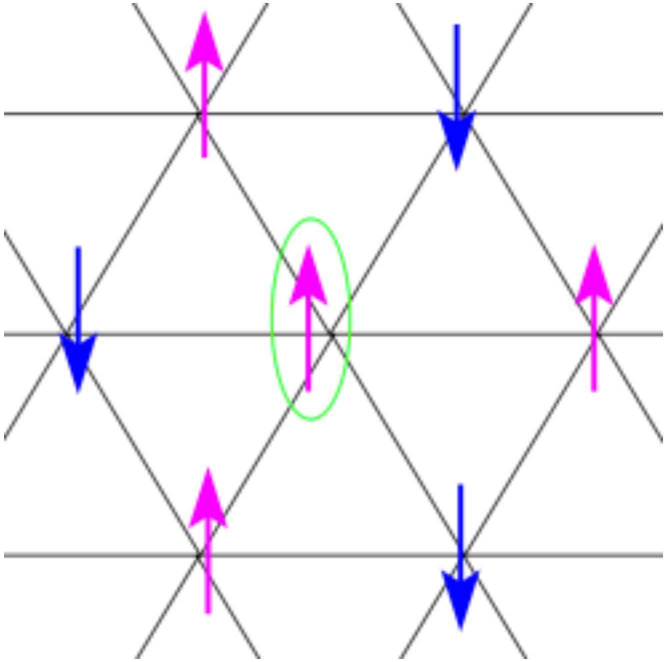
All interactions cannot be satisfied simultaneously

↓

One Bad Bond per Plaquette

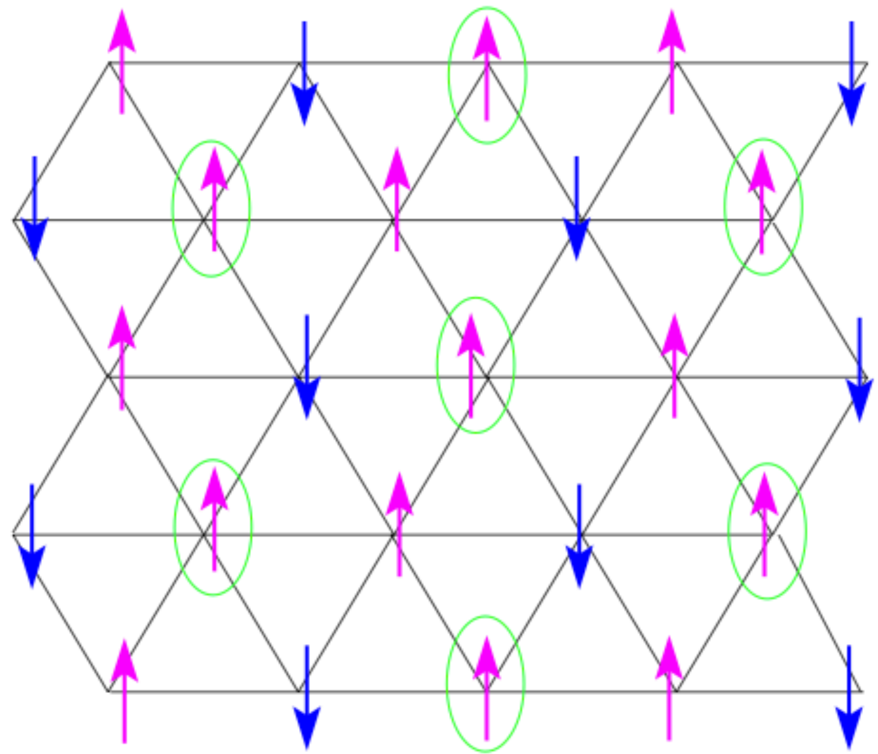
Frustration !!

"flippable spins" have zero exchange field



What happens at low T ???

Frustration



lower bound on entropy

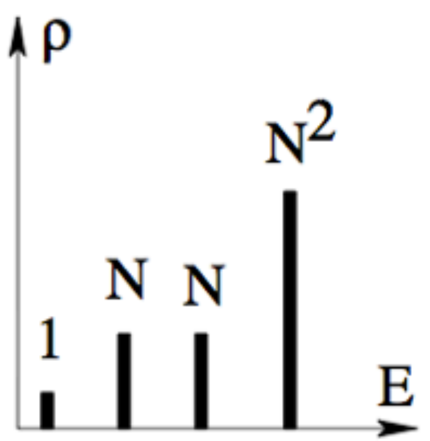
$$\mathcal{S} \geq (k_B/3)\ln 2$$

finite entropy in ground state: $\mathcal{S} = 0.323 k_B$ (Wannier, Houtappel)

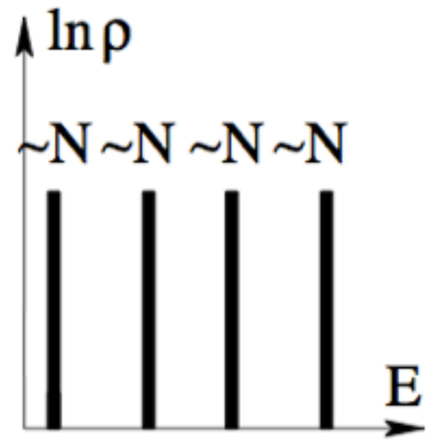
Large Ground-state Degeneracy Hallmark of Frustration
(Crucial: Local Degrees of Freedom)

Why degenerate systems are special

d.o.s – unfrustrated magnet



d.o.s – frustrated magnet



- Ground states can exhibit subtle correlations (seen at low T)
- Degenerate ground states provide no energy scale
  $\Rightarrow$ all perturbations are strong $\Rightarrow$ many instabilities
- Very rich behaviour (theory+experiment) – but also hard

(courtesy: R. Moessner)

Constraint counting as a measure of frustration

$$H = J\sum_{ij} S_i S_j \simeq (J/2)(\sum_{i=1}^{q} S_i)^2$$

gives ground state degeneracy:

$L \equiv \sum_i S_i$ to be minimised.

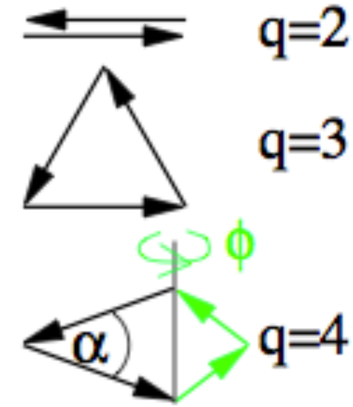
degeneracy grows with q

Constraint counting: $D = F - K$

- ground-state degeneracy D
- total d.o.f. F
- ground-state constraint K

Pyrochlore antiferromagnets are particularly frustrated

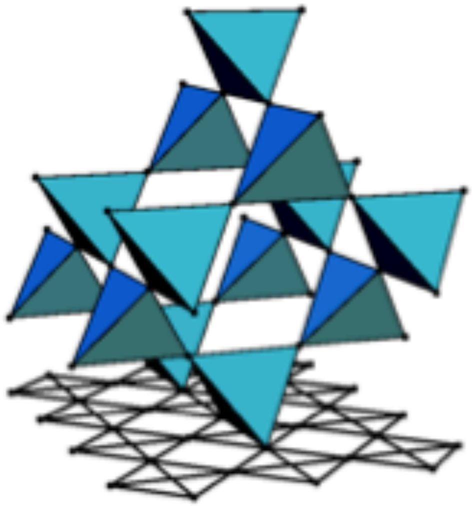
Units of q Heisenberg spins

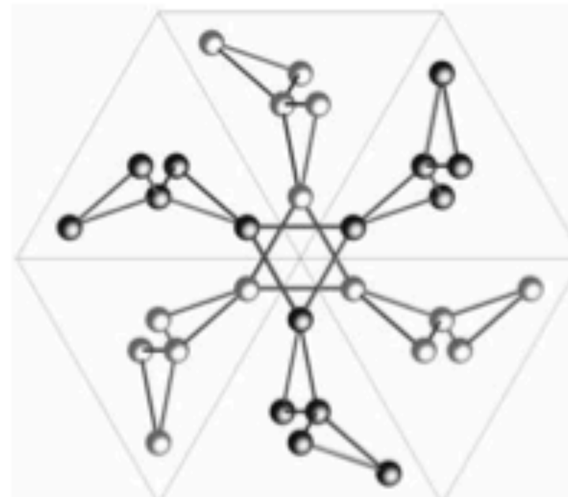


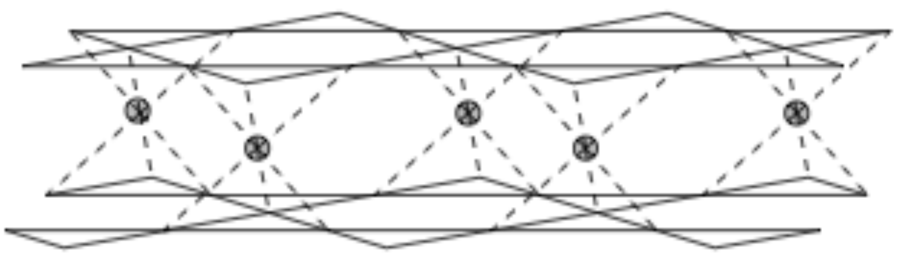
(Maxwell 1864)
(Chalker and Moessner 1998)

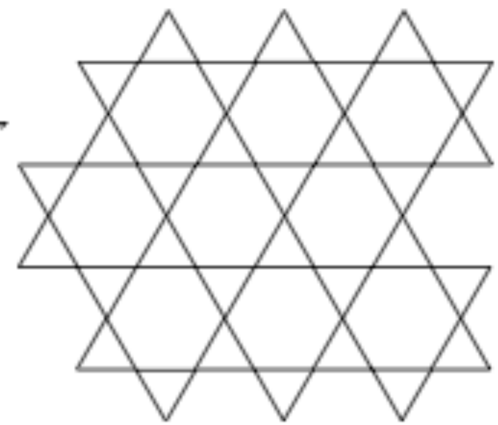
(courtesy: R. Moessner)

Highly frustrated (corner-sharing) lattices



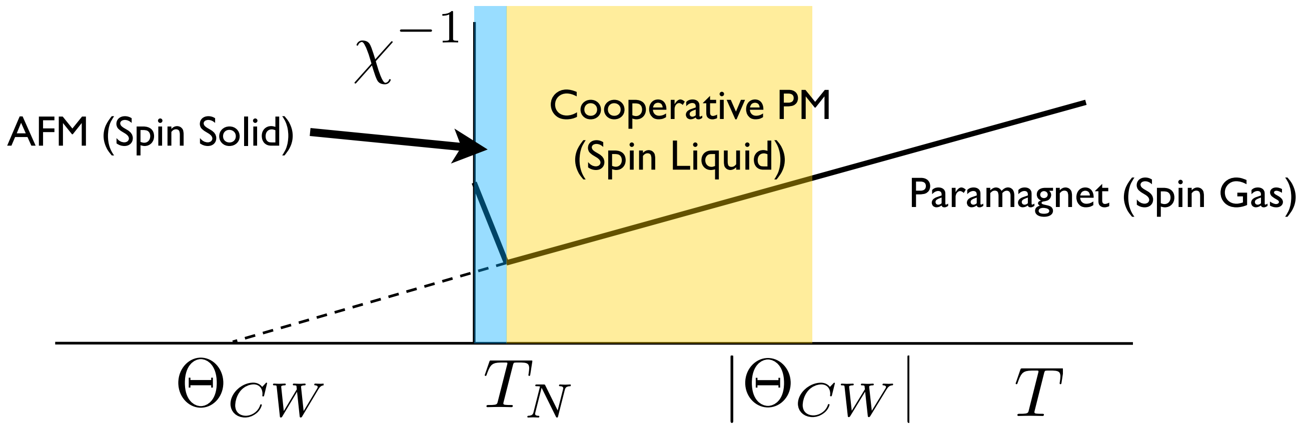






(courtesy: R. Moessner)

Experimental Signature



- Local moments: Curie-Weiss law at high T

$$\chi \sim \frac{A}{T - \Theta_{CW}}$$

- Frustration parameter: $f = |\Theta_{CW}|/T_N$ (Ramirez)

- $f >> 1$: wide regime $T_N < T < |\Theta_{CW}|$

(courtesy: L. Balents)

Nearest-neighbour spin ice

Anderson 1956

a toy model: the classical nearest-neighbour Ising antiferromagnet on the pyrochlore lattice:

$$\mathcal{H} = J \sum_{\langle ij \rangle} \sigma_i \sigma_j \sim \frac{J}{2} \left(\sum_{i=1}^{4} \sigma_i \right)^2$$

- energy minimised when $\sum_i \sigma_i = 0 \Rightarrow$ 2in-2out ice rules
- degeneracy: for a single tetrahedron $\binom{4}{2} = 6$ ground states

(courtesy: C. Castelnovo)

Zero-point entropy on the pyrochlore lattice

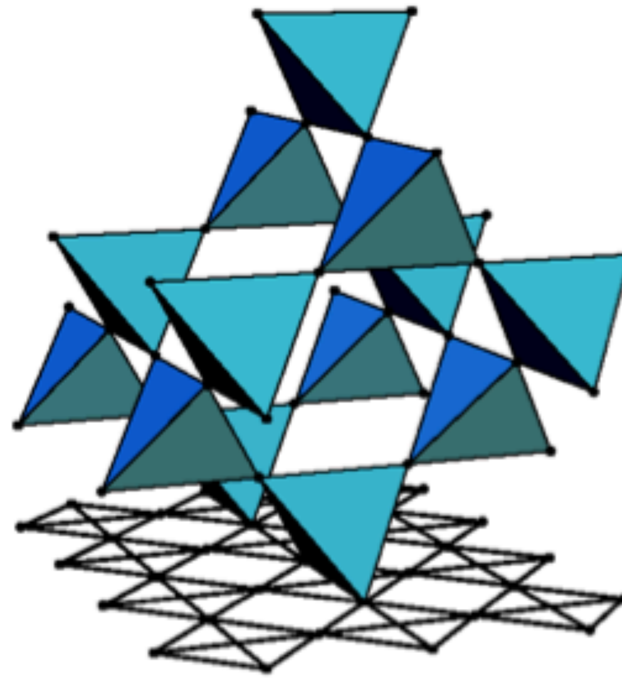
- Pyrochlore lattice = corner-sharing tetrahedra

$$\mathcal{H}_{\rm pyro} = \frac{J}{2} \sum_{\rm tet} \left(\sum_{i \in \rm tet} \sigma_i \right)^2$$

- Pauling estimate of ground state entropy $\mathcal{S}_0 = \ln N_{\rm gs}$:

$$N_{\rm gs} = 2^N \left(\frac{6}{16} \right)^{N/2} \Rightarrow \mathcal{S}_0 = \frac{N}{2} \ln \frac{3}{2}$$

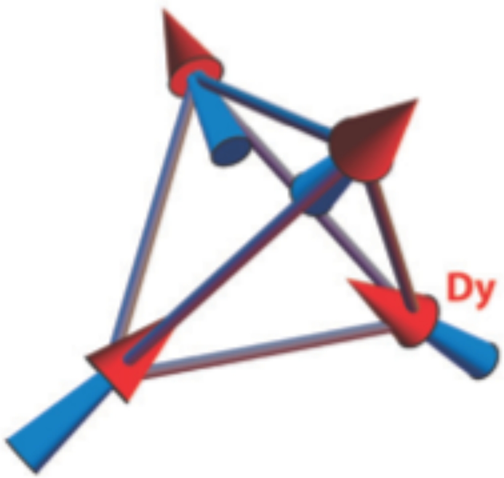
- microstates vs. constraints; N spins, $N/2$ tetrahedra

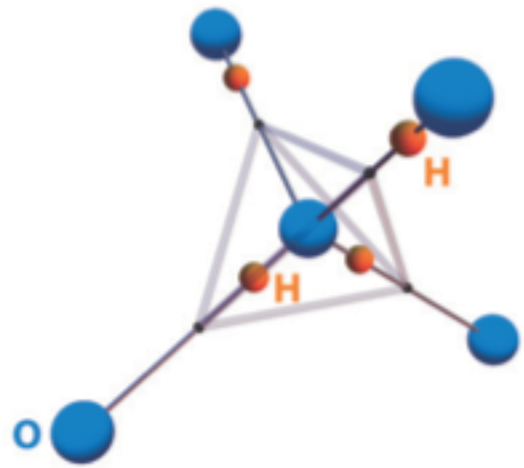


(courtesy: C. Castelnovo)

Mapping from ice to spin ice

- in ice, water molecules retain their identity
- hydrogen near oxygen ↔ spin pointing in





[Contribution from the Gates Chemical Laboratory, California Institute of Technology, No. 506]

The Structure and Entropy of Ice and of Other Crystals with Some Randomness of Atomic Arrangement

By Linus Pauling

Investigations of the entropy of substances at low temperatures have produced very important information regarding the structure of crystals, covery of the hydrogen bond[4] that the unusual properties of water and ice (high melting and boiling points, low density, association, high di-

(Pauling 1935)

Rutgers
Center for Materials Theory

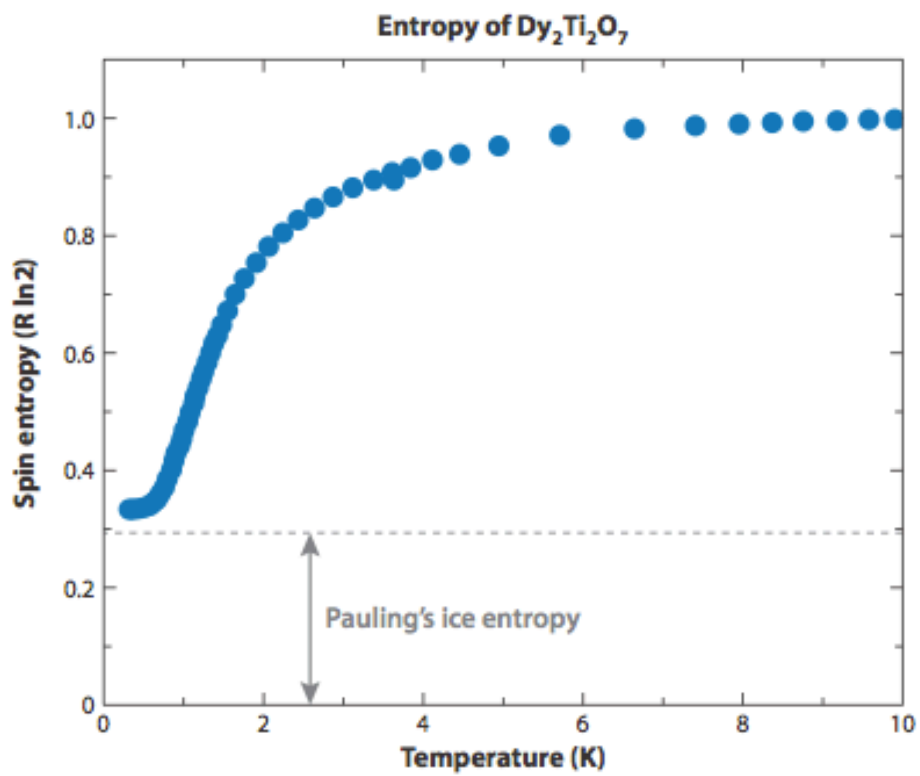
Spin Ice ($Dy_2Ti_2O_7$ and $Ho_2Ti_2O_7$)

Harris + Bramwell 1997

- local [111] crystal field $\sim$ 200 K
  $\Rightarrow$ Ising spins
- large spins (15/2 and 8)
  $\Rightarrow$ classical limit (small exchange $\sim$ 1 K)
- large magnetic moment $\sim 10\,\mu_B$
  $\Rightarrow$ long range dipolar interactions

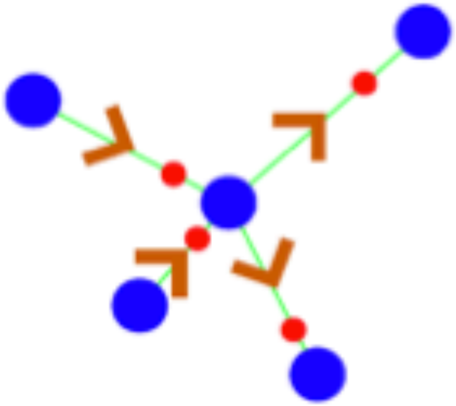
2-in, 2-out ice rules $\Rightarrow$ local constraint

Gingras *et al.* , Shastry *et al.* 1999-2001



Ramirez et al. (99)

Is spin ice ordered or not?



No order as in ferromagnet

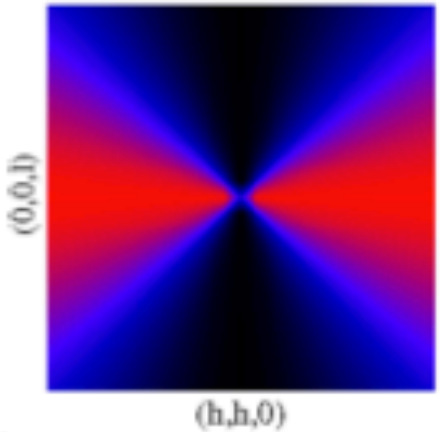
- extensive degeneracy

Not disordered like a paramagnet

- ice rules ⇒ 'conservation law'

Consider magnetic moments $\vec{\mu}_i$ as a (lattice) 'flux' vector field

- Ice rules ⇔ $\nabla \cdot \vec{\mu} = 0 \;\Rightarrow\; \vec{\mu} = \nabla \times \vec{A}$
- Simplest assumption: free field
  $\mathcal{S} = (K/2) \int |\nabla \times A|^2 \, dr^3$
- Local constr. ⇒ emergent gauge struct.
  - → algebraic spin corr. $\sim \frac{3\cos^2\theta - 1}{r^3}$
  - → structure factor (saddle point)

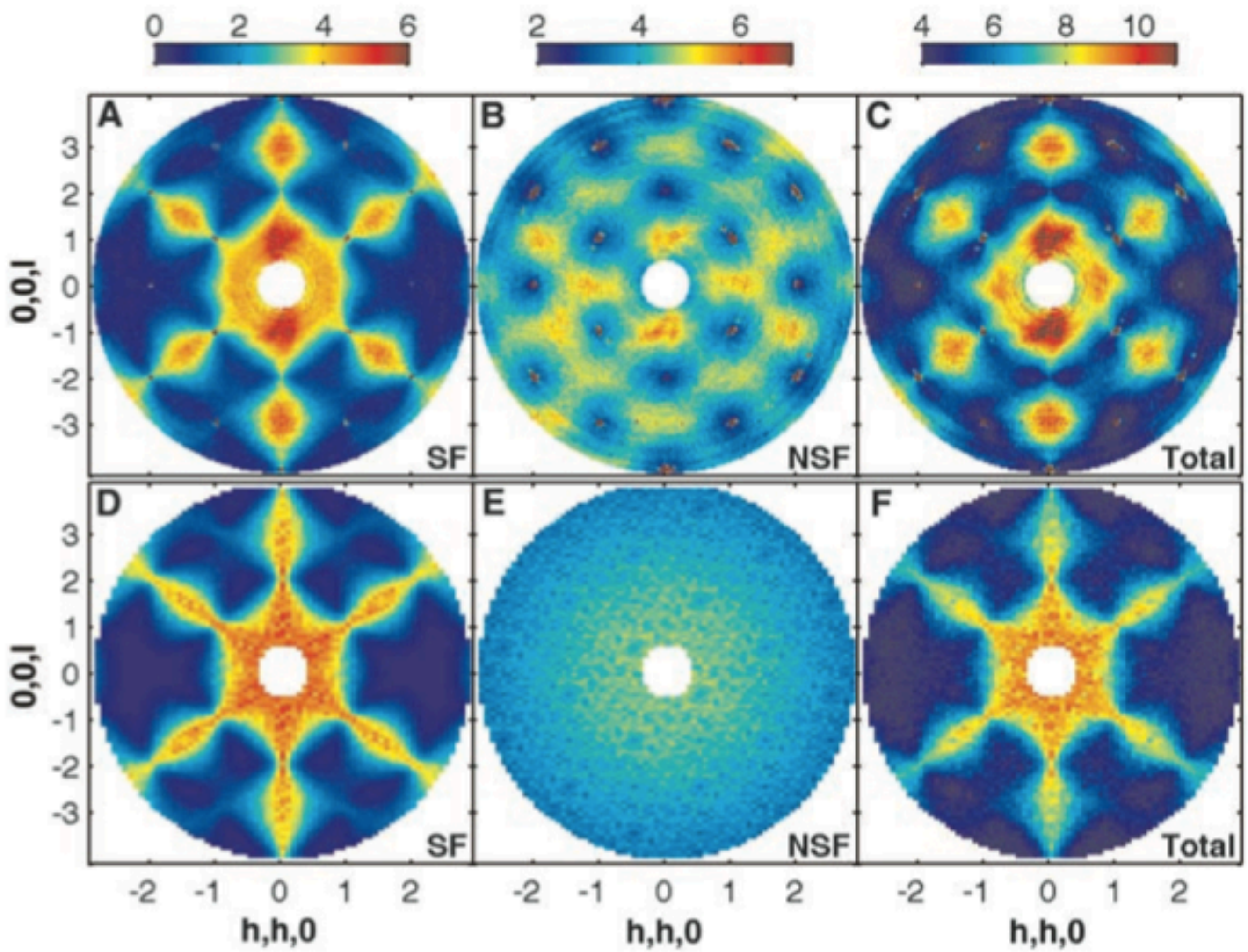


(courtesy: C. Castelnovo)

Magnetic Coulomb Phase in the Spin Ice $Ho_2Ti_2O_7$

T. Fennell,[1]* P. P. Deen,[1] A. R. Wildes,[1] K. Schmalzl,[2] D. Prabhakaran,[3] A. T. Boothroyd,[3] R. J. Aldus,[4] D. F. McMorrow,[4] S. T. Bramwell[4]

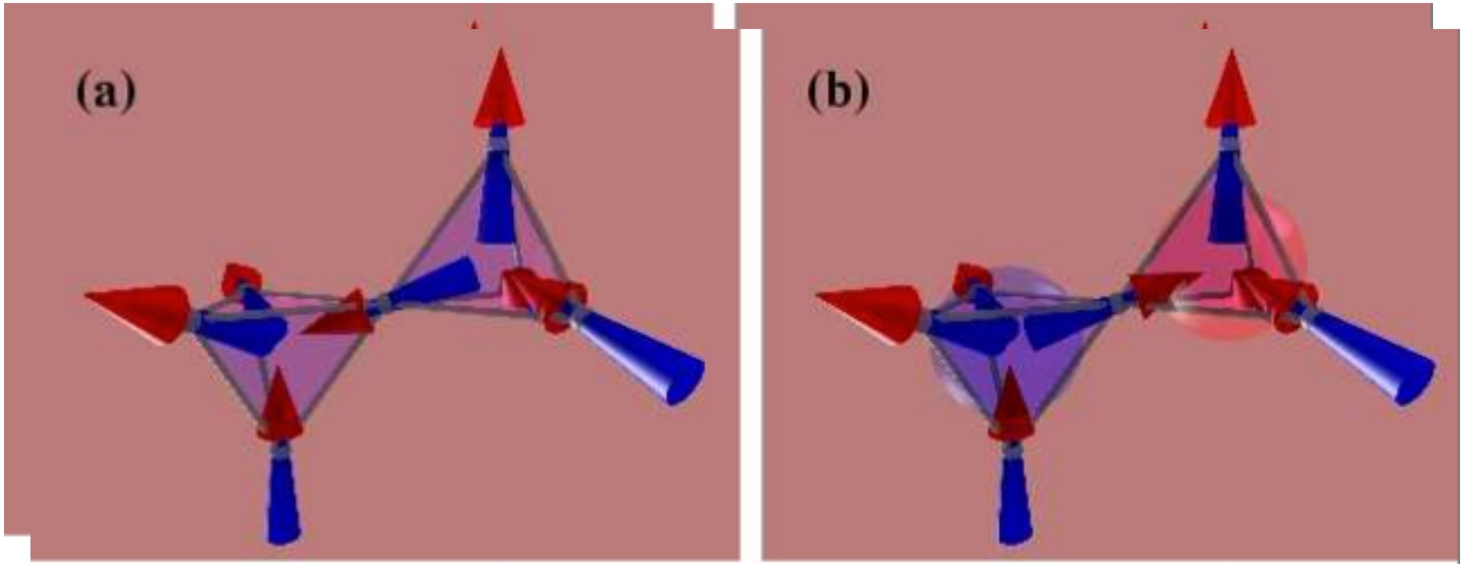
(2009)



Experiment

Theory

Excitations in Spin Ice



Journal of Experimental and Theoretical Physics, Vol. 101, No. 3, 2005, pp. 481–486.
Translated from Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, Vol. 128, No. 3, 2005, pp. 559–566.

ORDER, DISORDER, AND PHASE TRANSITIONS IN CONDENSED SYSTEMS

Magnetic Relaxation in Rare-Earth Oxide Pyrochlores

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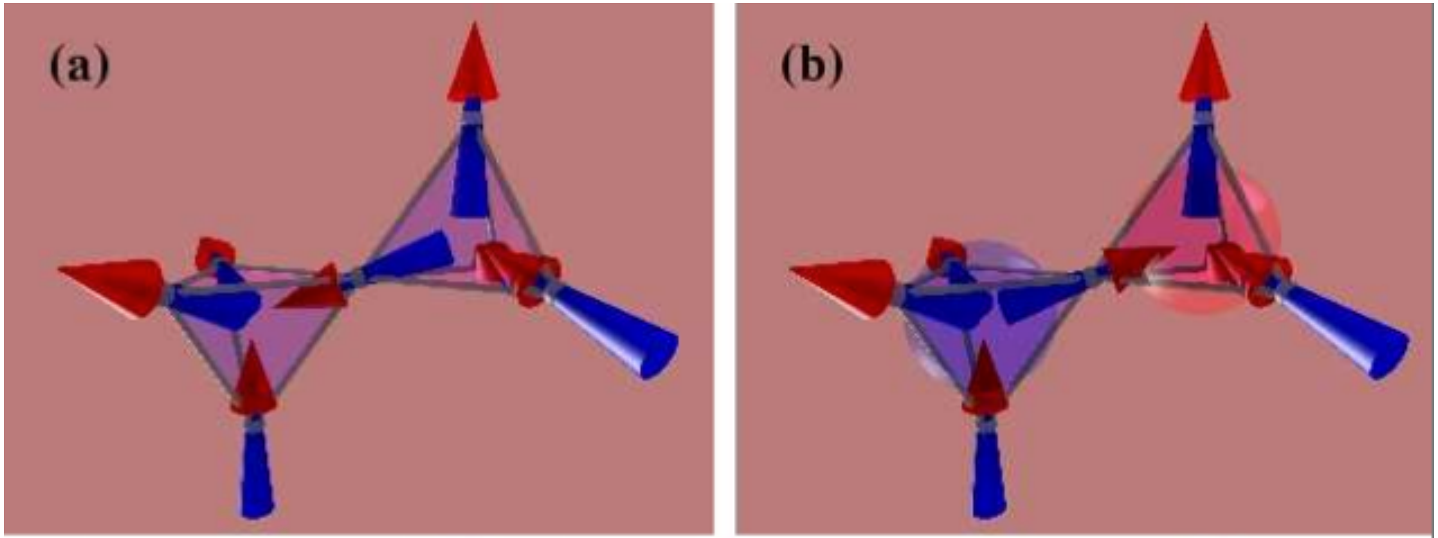
Received December 16, 2004

Excitations in Spin Ice



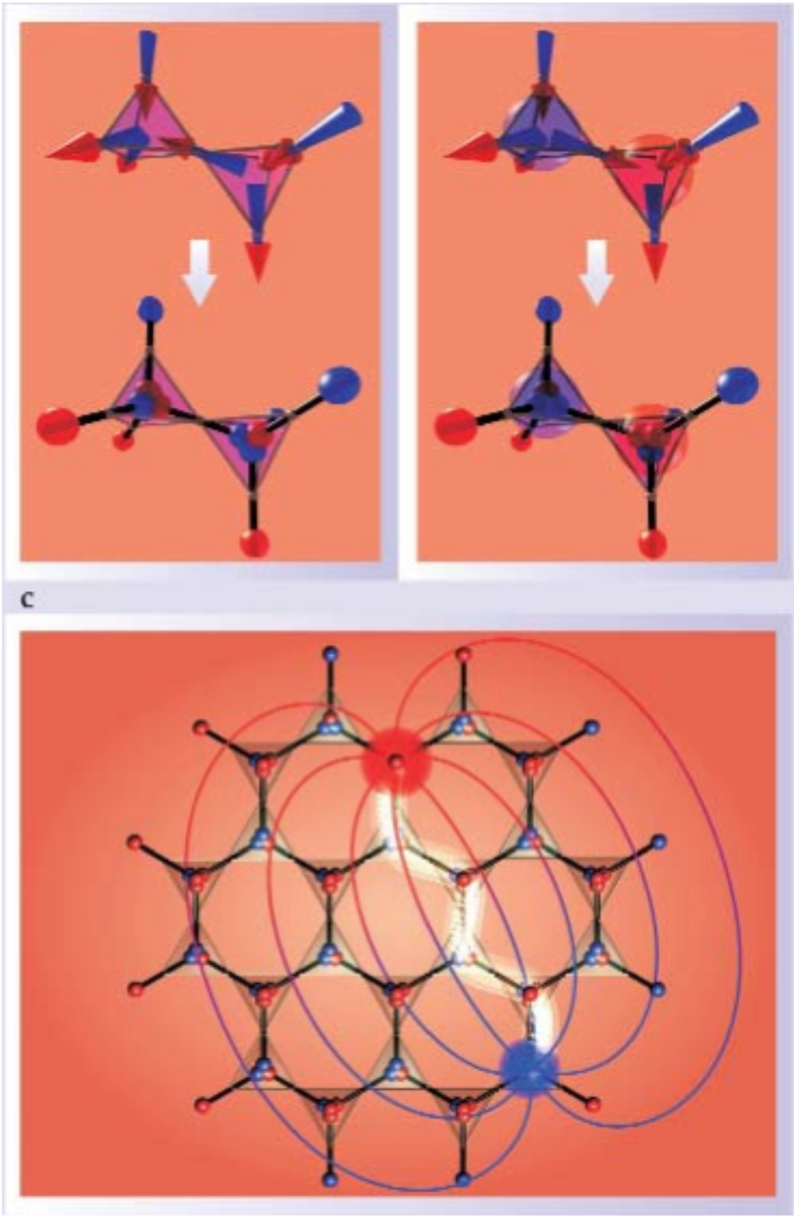
nature Vol 451 | 3 January 2008 | doi:10.1038/nature06433

LETTERS

Magnetic monopoles in spin ice

C. Castelnovo[1], R. Moessner[1,2] & S. L. Sondhi[3]

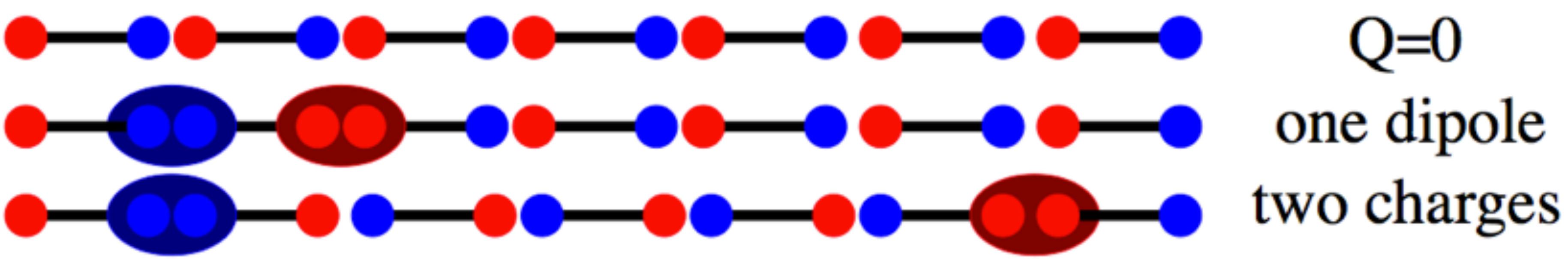
Visualization of the Dipole to Dumbbell Mapping



(Castelnovo, Moessner and Sondhi, 2008)

Excitations: dipoles or charges?

- Ground-state
  - no net charge
- Excited states:
  - flipped spin $\leftrightarrow$ dipole excitation
  - same as two charges?



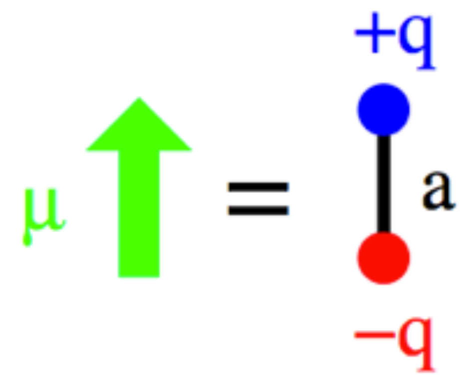
Fractionalisation in $d = 1$

(courtesy: C. Castelnovo)

The 'dumbell' model

Dipole $\approx$ pair of opposite charges ($\mu = qa$):

- Sum over dipoles $\approx$ sum over charges:



$$\mathcal{H}_{ij} = \sum_{m,n=1}^{2} v(r_{ij}^{mn})$$

- $v \propto q^2/r$ is the usual Coulomb interaction (regularised):

$$v(r_{ij}^{mn}) = \begin{cases} \mu_0\, q_i^m q_j^n / (4\pi r_{ij}^{mn}) & i \neq j \\ v_o(\frac{\mu}{a})^2 = \frac{J}{3} + 4\frac{D}{3}(1 + \sqrt{\frac{2}{3}}) & i = j, \end{cases}$$

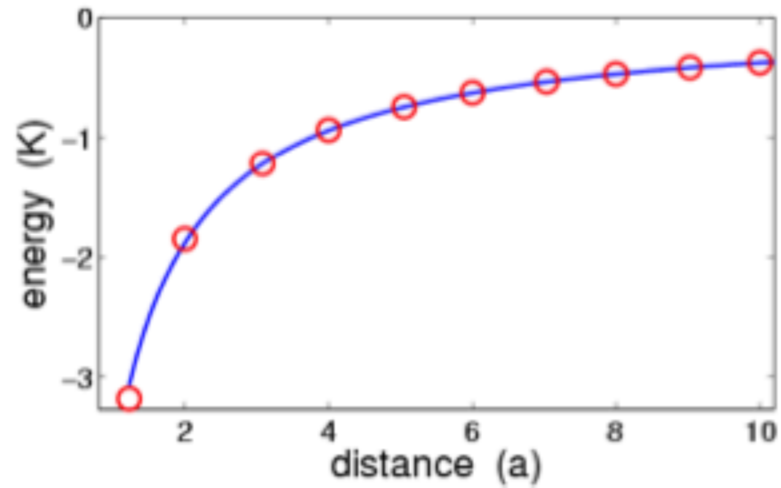
(courtesy: C. Castelnovo)

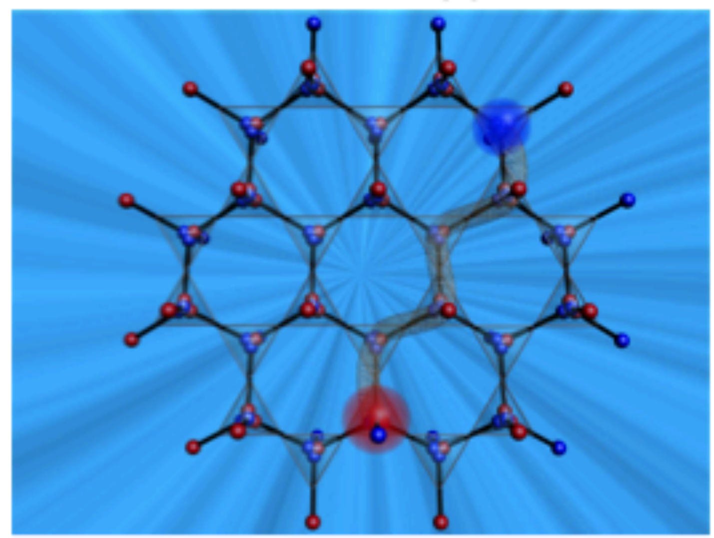
Deconfined magnetic monopoles

Dumbell Hamiltonian gives

$$E(r) = -\frac{\mu_0}{4\pi}\frac{q_m^2}{r}$$

- magnetic Coulomb interaction
- deconfined monopoles
  - charge $q_m = 2\mu/a = (2\mu/\mu_b)(\alpha\lambda_C/2\pi a_d)q_D \approx q_D/8000$
  - monopoles in H, not B





(courtesy: C. Castelnovo)

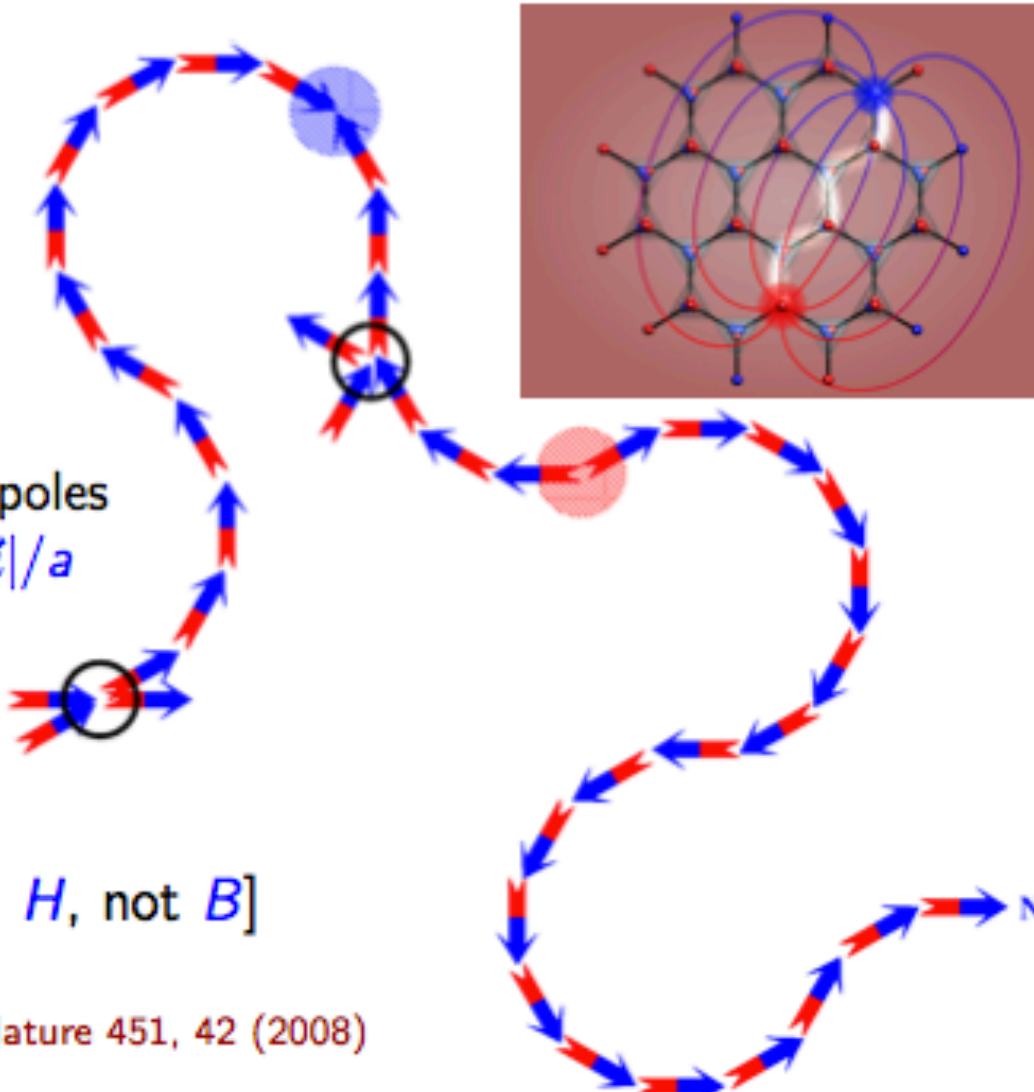
Elementary excitations: *emergent* magnetic monopoles

magnetic Coulomb interaction

$$E(r) = -\frac{\mu_0}{4\pi}\frac{q_m^2}{r}$$

- **deconfined** monopoles
- charge $q_m = \pm 2|\vec{\mu}|/a$

S N → S



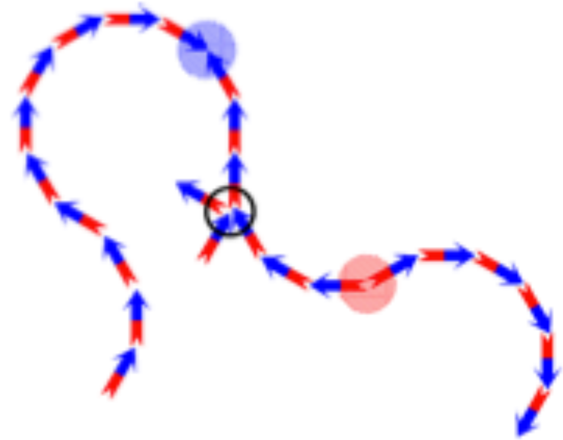
[monopoles in H, not B]

CC, Moessner, Sondhi, Nature 451, 42 (2008)

(courtesy: C. Castelnovo)

Magnetic monopoles? $\nabla \cdot \vec{M}$ vs. $\nabla \cdot \vec{H}$

no violation of $\nabla \cdot \vec{B} = 0$



- $\vec{B} = \vec{H} + \vec{M}$
- $\vec{M}$ is confined to the spins
- where a 'Dirac string' ends: $\nabla \cdot \vec{M} \neq 0$

$\Rightarrow$ defective tetrahedra ($\nabla \cdot \vec{M} \neq 0$) are sources and sinks of the magnetic field $\vec{H}$: $\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$

Unique setting!

(i) rare instance of fractionalisation in 3D

(ii) magnetic charges and network of 'Dirac strings' in 3D!

(iii) sources and sinks of magnetic field $\Rightarrow$ the monopoles couple to external probes (e.g., muons, SQUIDs, NMR-active nuclei)

(courtesy: C. Castelnovo)

Summary

Intro to Frustrated Magnets

- Large Ground-state Degeneracy is a Key Feature
- Maxwellian Constraint Counting Scheme
- Experimental Signature

Spin Ice

- Magnetic Analogue of Water Ice with “Ice Rules”
- Artificial Magnetostatics and Emergent Gauge Field
- Magnetic Coulomb Phase with Magnetic Monopoles
- Experiment !!
- Out-of-equilibrium dynamics?? Quantum Spin Ice ??