Home work #5

① Given

$$|-\rangle = \begin{pmatrix} \sin\left(\frac{\theta}{2}\right) e^{-i\varphi} \\ -\cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$|+\rangle = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) e^{-i\varphi} \\ \sin(\theta/2) \end{pmatrix}$$

a) Prove that the expressions above indeed represent eigenstates obeying $H|\underline{\pm}\rangle = \pm h |\underline{\pm}\rangle$ for $H = \bar{h}\cdot\sigma = \sum_{j=1}^{'} h_j \sigma_j$

b) Show that the spin polarization is parallel (antiparallel) to the magnetic field:

$$\langle \pm | \bar{\sigma} | \pm \rangle = \pm \bar{h}/h$$

2

(2) Prove that Berry curvature for adiabatic transport of the states:

$$|-\rangle = \begin{pmatrix} \sin(\theta/2)\, e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix} e^{i\Theta_-(\theta,\varphi)}$$

$$|+\rangle = \begin{pmatrix} \cos(\theta/2)\, e^{-i\varphi} \\ \sin(\theta/2) \end{pmatrix} e^{i\Theta_+(\theta,\varphi)}$$

{where $\Theta_\pm(\theta,\varphi)$ are smooth functions.}

is independent of the gauge choice determined by the arbitrary function $\Theta_\pm(\theta,\varphi)$.

3

(3) Calculate the Berry curvature $\omega_{\theta\varphi}$ of the state $|+\rangle$ and show that it is the opposite of that of $|-\rangle$; thus the sum of the Berry curvatures of these two states is zero everywhere in the parameter state.