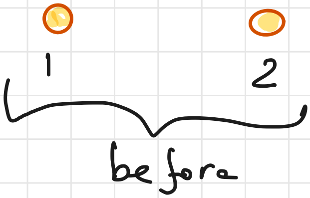
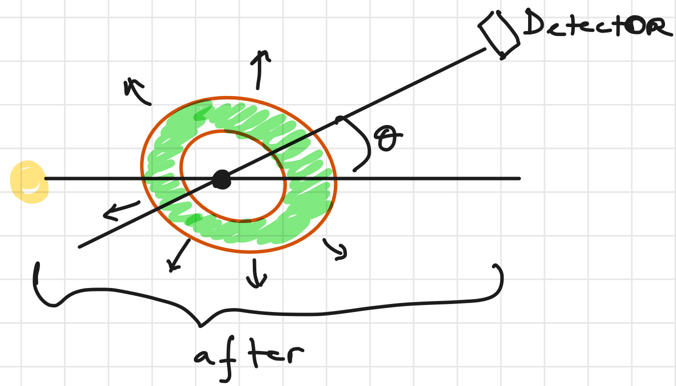
Identical Particles in QM

ORIGIN OF THE DIFFICULTY.

In QM particles have no definitive trajectory. ⇒ two separate w.p. at $t_0 = 0$ will mix up at time t. As the result we will lose the track of particles. ⇒ we will have no way to determine which particle we will detect with a probability p.

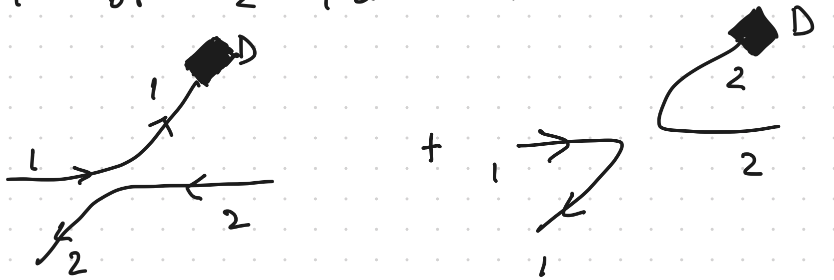
Lets investigate this point in detail by going back to our scattering problem from the last lecture.





here two w.p. overlap during the collision and the green shell is where the probability is $\neq 0$

Now, we place the detector D at the angle θ w.r.t to initial velocity direction.
Now we have a problem what particle 1 or 2 the Detector catches?

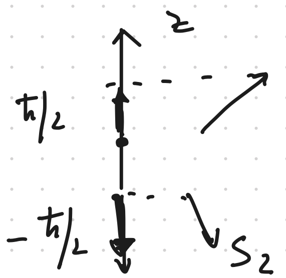


As the particles are identical we cannot say what's the final state w.f.

So what's the problem?

Above we tried to label the particles as 1 and 2 but failed to determine the ket for a given result.
but the same problem is also related to the initial ket. known as the exchange degeneracy.

To explain I will switch to somewhat simpler example.
Assume you have 2 spin particles and we made a complete measurement on the spins, so we know the total spin.



S_1 and S_2 are two spin observables.

and $|\varepsilon_1\, \varepsilon_2\rangle$ $\quad \varepsilon_1$ and $\varepsilon_2 = +$ or $-$

is the basis of the state space formed by the common eigenkets S_{1z} and S_{2z}

↓

eigenvalues

$\varepsilon_1 \hbar/2$ $\quad \varepsilon_2 \hbar/2$

Reminder for distinguishable particles we can write
$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \dots |\psi_N\rangle$

Now we can associate 2 states with the physical state.

This is just an idea! But is it correct?

(*) $|\varepsilon_1 = +, \varepsilon_2 = -\rangle = |+\rangle \otimes |-\rangle$ with any vector in this subspace

$|\varepsilon_1 = -, \varepsilon_2 = +\rangle$,

if it's correct $\Rightarrow$

all $|\text{kets}\rangle$ here $\longleftarrow$ $|\psi\rangle \equiv \begin{cases} \alpha|+,-\rangle + \beta|-,+\rangle \\ |\alpha|^2 + |\beta|^2 = 1 \end{cases}$

represent the same physical state as in (*)

or spin up and spin down

This is called exchange degeneracy

Here is the problem: Let's ask what's probability a component of both spins along Ox being equal?

The spin operator for a particle in any direction :

$$\hat{S} = \frac{\hbar}{2} \hat{\sigma}$$

$\sigma = (\sigma_x , \sigma_y , \sigma_z)$, The eigenvectors of

$\hat{S}_z$: $|S_z = +1/2\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|S_z = -\frac{1}{2}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

and S_x : $|S_x = +1/2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \left\{ |S_z = +1/2\rangle + |S_z = -\frac{1}{2}\rangle \right\}$

$|S_x = -1/2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ +1 \end{pmatrix} = \frac{1}{\sqrt{2}} \left\{ -|S_z = 1/2\rangle + |S_z = -\frac{1}{2}\rangle \right\}$

The state vector associated wit both spins in x and both $+\frac{1}{2}$:

$$|S_{1x} = +\tfrac{1}{2}\rangle \otimes |S_{2x} = +\tfrac{1}{2}\rangle \equiv |\Psi_0\rangle \equiv$$

$$\equiv \frac{1}{\sqrt{2}} \left(|S_{1z} = \tfrac{+1}{2}\rangle + |S_z = -\tfrac{1}{2}\rangle \right) \otimes \frac{1}{\sqrt{2}} \left(|S_{z_2} = +1/2\rangle + |S_{z_2} = -1/2\rangle \right) = \frac{1}{2} \left(|+\tfrac{1}{2} \ 1/2\rangle + |+1/2 \ -\tfrac{1}{2}\rangle + |-\tfrac{1}{2} \ +\tfrac{1}{2}\rangle + |-1/2 \ -1/2\rangle \right) = |S_{1x} = +1/2 , S_{2x} = +1/2\rangle$$

Now considering that our state:

$$|\psi\rangle = \alpha \, |\tfrac{1}{2} \ -\tfrac{1}{2}\rangle + \beta \, |-\tfrac{1}{2} \ \tfrac{1}{2}\rangle$$

The probability of obtaining the result of $+\frac{1}{2}$: or $\langle \Psi_0 | \psi \rangle$?

$$P = |\langle \Psi | \underbrace{S_{1x} = +1/2 \;\; S_{2x} = +\tfrac{1}{2} \rangle}_{|\Psi_0\rangle \equiv}|^2 =$$

$$\langle \Psi | = \alpha^* \langle 1/2 \; -1/2 | + \beta^* \langle -1/2 \, , 1/2 | \quad S_{1x} = \tfrac{+1}{2} \; S_{2x} = \tfrac{+1}{2} \equiv |\Psi_0\rangle$$

$$\tfrac{1}{2} (|+1/2 \; +1/2 \rangle + |\tfrac{+1}{2} \; -\tfrac{1}{2} \rangle + |-\tfrac{1}{2} \; +1/2 \rangle + |\tfrac{+1}{2} \; -1/2 \rangle =$$

$$P \equiv |\tfrac{1}{2} (\alpha + \beta)|^2 =$$

$$= \tfrac{1}{2} \; |\alpha^2| + |\beta|^2 + 2 Re(\alpha \beta^*) = \;=\; Re(\alpha \beta^*) + 1/2$$

This probability depend on on the choice of α and β!

To remove this ambiguity we need to specify which vector is to be chosen i.e. the exchange degeneracy should be removed.

In short we have a problem of the mathematical description of the initial state in the scattering as well, and cannot solve the problem of degeneracy of identical particles declaring that all the states are the same!.

LECTURE

Identical Particles

on the way to quantum field theory

Particles are identical when we cannot distinguish them based on any physical property.

- Many-body physics
- pure quantum phenomena

In classical mechanics we can track any particle individually. Since for quantum we deal with the wave like properties of amplitudes of probability we cannot distinguish them any longer.

e.g.

so unless we consider the case when 2 particles are well separated, i.e. without the overlap we cannot talk any longer about the individual objects in QM

We need new formalism. As usual the best way to deal with this situation is to consider symmetry for the ansemble of the quantum particles

① Permutation symmetry

if we consider a system below the threshold for getting it into the excided state we have 4 - degrees of freedom i.e.

$\bar{p}$, $\bar{r}$, $\bar{\sigma}$ and $\varepsilon \Rightarrow$ helicity
spin

Consider a system which has N-particles

$1 \equiv \{\bar{p}_1 \ \bar{r}_1 \ \sigma_1\}$

$2 \equiv \{\bar{p}_2 \ \bar{r}_2 \ \sigma_2\}$

etc.

Hamiltonian can be written as:

$H(1,2,3 \ldots N)$ and so is $\Psi(1,2, \ldots N)$

Lets introduce a new operator P_{ij} = the permutation operator $i \leftrightarrow j$ with eigenvalue ω.

$$P_{ij}\Psi(1,2\ldots, i, \ldots j \ldots N) = \omega \Psi(1,2, \ldots i, \ldots j \ldots N)$$

do this twice

$$P_{ij}^2 \ \Psi(\ldots i, \ldots j \ldots) = \omega P_{ij} \Psi(\ldots j \ldots i \ldots) =$$

$$= \omega^2 \Psi(\ldots i \ldots j \ldots) \Rightarrow \omega^2 = 1 \text{ or } \omega = \underline{\pm 1}$$

$\uparrow$ real

then P_{ij} is a hermitian operator.

$$P_{ij}^2 = I \Rightarrow P_{ij} = P_{ij}^{-1}$$

also you can show that P_{ij} is unitary.

i.e. $U O U^{-1} = O'$

e.g. $P H P^{-1} = H \Rightarrow PH = HP$ and for any symmetric operator O we get $[P, O] = 0$

if $\Psi(1 \ldots N)$ is eigenstate of H

then $P H \Psi = P E \Psi = E \underline{P\Psi}$ is also an eigenstate

THIS IS CALLED EXCHANGE DEGENERACY

also <u>all observables</u> are the same for Ψ and $P\Psi$ ← we discussed this for spins in detail above

Now we have $N!$ exchange operators for N particles, there has to be at least one eigenfunction such as:

$$H\Psi = E\Psi \quad \text{AND} \quad P_{ij}\Psi = \underset{=\pm 1}{\omega_{ij}}\Psi$$

also one can show that $P_{ij}P_{ik} = P_{jk}P_{ij} = P_{ik}P_{jk}$

see PROBLEM 1 ch. 18

This also means

$$P_{ij} P_{ik} \Psi = w_{ij} w_{ik} \Psi \Rightarrow$$

$$w_{ij} w_{ik} = w_{jk} w_{ij} = w_{ik} w_{jk}$$ ← those are just numbers

which gives $w_{ik} = w_{jk}$ $\qquad w_{ij} = w_{ik}$

or $w_{ij} = w_{ik} = w_{jk}$

if $w_{ij} = 1$ then Ψ is symmetric under interchange of any pair of particles $\equiv \Psi_s(1 \dots N)$

if $\Psi_{ij} = -1$ Ψ antisymmetric $\equiv \Psi_a(1 \dots N)$

$$P_{ij} \Psi_s(1 \dots i \dots j \dots N) = \Psi_s(\dots j \dots i) = +1 \Psi_s(i \dots j)$$

$$P_{ij} \Psi_a(i \quad j) = -\Psi_a(i \dots j)$$

Also: • $P \Psi_a = \Psi_s$

$$P \Psi_a = (-1)^P \Psi_a \qquad (-1)^P = \begin{cases} 1 & \text{even} \\ -1 & \text{odd} \end{cases} \text{ permutations}$$

also we can write down:

$$P\Psi = e^{i\theta} \Psi \qquad \text{where } \theta = 0, \text{ or } \pi$$

if θ is any $0 < \theta < \pi$ the particle is called $\boxed{\text{anyon}}$

very important for quantum computing!

e.g. Consider two partices $N = 2$

P_{12} is the only operator

for 3-particles $P_{ij} \Psi(1, 2, 3)$ $\qquad P_{12}$
$i = 1, 2, 3$ $\qquad P_{13}$
$j = 1, 2, 3$ $\qquad P_{23}$
$\qquad P_{13}$

$\underline{P_{12} \cdot P_{13}} \Psi =$
$= P_{12} \Psi(3\,2\,1) =$
$= \Psi(3\,1\,2)$

$\underline{P_{13} P_{12}} \Psi(123) =$
$= P_{13} \Psi(213) =$
$= \Psi(231)$

don't commute!
so there are states where all P_{ij}'s are not diagonal.

However, experimentally we know that only $\theta = 0$ and $\theta = \pi$ are realized.

- the Hilbert space is broken into subspaces H_s and H_a and the remaining unphysical $(N!-2)$ states H_r.

! { Since P commute with H the symmetric characters cannot be changed over time.
So fermions remain frmions → bosons remain bosons.

- Moreover, if we have a syster of identical particles they will be described by a uniquely symmetrical wave f., otherwise the states would mix between symm. and anti-symm.

Spin - statistics theorem.

- A wave function of N identical partices of $\frac{1}{2} \cdot n$ where n is odd must have antisymmetric wave function under exchange of any two particles.
- Wave function of any N particles with n where n is even or "0" must be symmetric.

So there are 2 quantum statistics.
Bose - Einshtein $n =$ even or 0 ⇒ bosons
Fermi - Dirac $n = \frac{1}{2} m$ $m \equiv$ odd. ⇒ fermions

it works even for composite partices
e.g.
$^3He = 2$ protons + neutron
$2 \cdot \frac{1}{2} + \frac{1}{2} = 1\frac{1}{2}$ = fermion

^4He = boson = and the subject to B-E condensation = superfluitity

Symmetry of w.f.

Many-body Sch. eqn.

$$i\hbar \frac{\partial}{\partial t} \Psi(1 \ldots N) = H(1 \ldots N)\, \Psi(1 \ldots N)$$

Among all possible solutions we need to construct at least either symm. or antisymm. w.f.

Let's see how it's done. For Ψ_s:

Recall among $N!$ solutions corresponding the same energy eigenvalue $P\Psi$. So we can sum them up and then normalized.

This can be understood that if we have a bunch of $\Psi(1 \ldots N)$ $P\Psi \to$ will just transform one of them into another which is includes into this sum.

how to Ψ_a: So the antisymm. can be setup as a sum of all permuted w.f. by means of even interchanges of pairs and substracting the sum of all the permuted functions constructed by mean of odd number of interchanges.

Example: 2 particles: Lets assume $\Psi(12)$ is the solution of Sch. eqn.

then $P_{12}\Psi(12) = \Psi(21)$ is also a solution

$$\Psi_s = A\left(\Psi(12) + \Psi(21)\right) \qquad \Psi_a = B\left(\Psi(12) - \Psi(21)\right)$$

$$P\Psi_s = A\left(\Psi(21) + \Psi(12)\right) = \Psi_s \qquad \Psi_a = B\left(\Psi(21) - \Psi(12)\right) = -B\,\Psi_{12}$$

$$A = B = \frac{1}{\sqrt{2!}}$$

For N particles

$$\boxed{\begin{aligned}\Psi_s &= S_s\,\Psi(1\ldots N,t) = \frac{1}{\sqrt{N!}}\sum_P (1)^P P\,\Psi(1\ldots N,t)\\ \Psi_a &= S_a\,\Psi(1\ldots N,t) = \frac{1}{\sqrt{N!}}\sum_P (-1)^P P\,\Psi(1\ldots N,t)\end{aligned}}$$

Ex. 3 particles: $\Psi(1\,2\,3)$ $\quad N! = 3! = 1\cdot 2\cdot 3 = 6$

$$\Psi_s = \frac{1}{\sqrt{6}}\big(\Psi(123) + \Psi(213) + \Psi(132) + \Psi(321) + \Psi(312) + \Psi(231)\big]$$

even # of permutation

$$\Psi_a = \frac{1}{\sqrt{6}}\big(\Psi(123) - \Psi(213) + \Psi(231) - \Psi(132)$$

odd number

$p=1 \Rightarrow (-1)^1$ $\quad p=2 \;\; (-1)^2$ $\quad$ and so on.

$$- \Psi(321) + \Psi(312)\big]$$

Note for $\Psi_s^* \Psi_s$ and $\Psi_a^* \Psi_a$ the w.f. doesn't change under permutation of 2 particles. So any measurable quantity is not sensetive to the particle exchange.

$$|\Psi_a|^2 = \frac{1}{2}\Big(\Psi(1,2)^2 + \Psi(21)^2 - \underbrace{\Psi(12)\Psi^*(21)}_{\text{introction term!}}\Big)$$

Not true for anyons ⇒ BRAIDING!

if particles are well separated this term = 0 and particles are distinguished.

e.g. for $\varepsilon \gg k_B T$ and low number of particles per quantum state is small and we can apply classical statistics

Sorry! for the quality

Pauli Exclusion Principle

[No two fermions can be in the same quantum state (i.e. have the same quantum numbers).]

It is not possible to solve a many-body problem exactly.

The way we solve 1) assume particles are non-interacting, 2) include interactions via perturbation theory.

Specifically:

$$H(1\ldots N) = H(1) + \ldots + H(N) \Rightarrow$$

$$\begin{cases} \Psi(1\ldots N) = \psi(1)\,\psi(2)\,\psi(N) \\ E_0 = E_{\alpha_1} + E_{\alpha_2} + \ldots E_{\alpha_N} \\ H_0(j)\,\phi_\alpha(j) = E_\alpha\,\phi_\alpha(j) \end{cases}$$

The eigenfunction corresponding to E_0 will be a linear combination of $\Psi(1\ldots N)$

In general Ψ_a can be written as determinant (Slater determinant)

$$\Psi_a = \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_{\alpha_1}(1) & \phi_{\alpha_2}(2) & \ldots & \phi_{\alpha_N}(N) \\ \phi_{\alpha_2}(1) & \cdot & & \vdots \\ \vdots & & \ddots & \vdots \\ \phi_{\alpha_N}(1) & \phi_{\alpha_N}(2) & \ldots & \psi_{\alpha_N}(N) \end{vmatrix}$$

$\phi_{\alpha_N}(j)$ — j th particle

with α_j wave functions.

Change of sign comes from the change of sign upon exchange of 2 columns.

By looking at the Slater matrix we can see also if any of two particles say 1 and 2 have the same α_{ij} => the determinant = 0

However for bosons any two or more particles can occupy the same state i.e. the occupation number for bosons 0, 1, 2 ... for fermion it is ether 0 or 1.

SPIN OF 2 ELECTRONS

$$H = \frac{1}{2m} \sum_{i=1}^{N} p_i^2 + V(r_1\ r_2 \ldots r_N)$$

has no spin operator or if SO is neglected.

→ $\Psi(r_1 \ldots r_N)$. IF we include spin we need to add the spin eigenfunction $\chi(r_1 \ldots r_N)$ so

$$\Psi(1 \ldots N) = \Phi(r_1 \ldots r_N) \cdot \chi(r_1 \ldots r_N)$$

or some sort of linear combo of such product $\phi \cdot \chi$. This is the 1st approx for SO interaction.

↑ spin-orbit

We still need to apply the same symmetry argument to the total wave function, but now we consider both χ and ϕ.

The issue is that we can construct asymm. and symm. parts for χ and ϕ separately.

$\left.\begin{array}{l} \text{sym} \times \text{sym} \\ \text{asym} \times \text{asym} \end{array}\right\} = \text{sym}$ ← only for bosons

$\text{sym} \times \text{asym} = \text{asym.}$ ← only possible for fermions

Lets recal how it works for spins

for angular moment $\bar{J}_1$ $|j_1 m_1\rangle$
J_2 $|j_2 m_2\rangle$

$\Rightarrow \bar{J} = \bar{J}_1 + \bar{J}_2$

using Glebsch - Gordon matrix

$|\psi\rangle = |j_1 m_1\rangle |j_2 m_2\rangle$

$(2j_1+1)(2j_2+1)$ eigenstates total

The same can be applied to spins:
$s = \frac{1}{2}$ $|\frac{1}{2}\ \frac{1}{2}\rangle \equiv \alpha$ and $|\frac{1}{2}\ -\frac{1}{2}\rangle \equiv \beta$

— The spin part for 2 electrons 1 and 2

$\alpha(1)\ \alpha(2)$
$\alpha(1)\ \beta(2)$
$\beta(1)\ \alpha(1)$
$\beta(2)\ \alpha(2)$

- Next we can construct 4 commuting operators out of S:

$$\left\{\begin{array}{l} S^2 = (S_1+S_2)^2 = S_1^2 + S_2^2 + 2S_1 \cdot S_2 \\ S_z = S_{1z} + S_{2z} \\ S_{1z}^2 \\ S_{2z}^2 \end{array}\right.$$

the common set of eigenstates we label as

$|S\, m_S\rangle$ where $s = \frac{1}{2} + \frac{1}{2} = 1$ $\quad m = -1, 0, 1$
$\frac{1}{2} - \frac{1}{2} = 0$

$$\left.\begin{array}{l} |0\,0\rangle \\ |1\,-1\rangle \\ |1\,0\rangle \\ |1\,-1\rangle \end{array}\right\}$$ = 4 eigenkets in this coupled representation

The Clebsch-Gordan matrix for this representation is:

$$\chi_s^{(1)} = |11\rangle = \alpha(1)\,\alpha(2)$$

$$\chi_s^{(0)} = |10\rangle = \frac{1}{\sqrt{2}}\left[\alpha(1)\beta(2) + \beta(1)\alpha(2)\right] \quad \checkmark \quad \uparrow\uparrow$$

$$\chi_s^{(-1)} = |1-1\rangle = \beta(1)\beta(2)$$

$$\chi_a = |00\rangle \; \uparrow\downarrow = \frac{1}{\sqrt{2}}\left[\alpha(1)\beta(2) - \beta(1)\alpha(2)\right]$$

See Ch 11.12 of the text about 2 moments if you forgot.

$$S^2 = (\underbrace{S_1 + S_2}_{\equiv S})^2 \cdot |S\,m_s\rangle = S(S+1)\,|S\,m_s\rangle$$

In units of $\hbar$

$$S_z\,|S\,m_s\rangle = m_s\,|S\,m_s\rangle$$

The eigenstates for S_z:

$$\chi_s^{(1)} = \hbar$$
$$\chi_s^{(0)} = 0$$
$$\chi_s^{(-1)} = -\hbar$$
$$\chi_a = 0$$

S^2 eigenvalue $= 2\hbar^2$

$\chi_s^{(1),(0),(-1)}$ = TRIPLET STATE AND SYMMETRIC

χ_a = ANTI-SYMM.

Remember this is only the spin part.

The total wave function $\chi \cdot \phi(r)$

STILL NEED TO BY ANTISYMM.

[SEE PROBLEM 3 very instructive] page 453.

Exchange Interaction.

In non relativistic version of Q.M. we have no notion that interaction may depend on spin.

Consider a system of 2 electrons.

$$H(r_1, r_2) = K_1(r_1) + K_2(r_2) + V(\underbrace{|r_2 - r_1|}_{=r})$$

$\boxed{\begin{array}{l}\bar{R} = \bar{r}_1 + \bar{r}_2 \\ \bar{r} = \bar{r}_1 - \bar{r}_2\end{array}}$

in the c.m representation $\quad \frac{1}{M} = \frac{1}{m_1} + \frac{1}{m_2} = \frac{m_1 + m_2}{m_1 m_2} = \frac{1}{\mu}$

$$H(r, R) = H_{rel}(r) + K_{cm}(R) = \frac{1}{2\mu} P^2_{rel}(r) + V(r)$$

$$+ K_{cm}(R) = \frac{1}{2M} P^2_{cm}(R)$$

P_{12} commutes with $H(r_1 r_2) \Rightarrow [P_{12}\ K_{cm}] = 0 =$

$$= [P_{12}\ H_{rel}] = 0$$

The eigenstates of K_{cm} and H_{rel} can be either symmetric or antisymm. under exchange:

e.g. for K_{cm}: $e^{iK_{cm}R} = e^{iK_{cm}(r_1 + r_2)}$

$$P_{12}\, e^{iK_{cm}(r_1 + r_2)} = e^{iK_{cm}(r_2 + r_1)}$$

$=$ symmetric!

So the overall symmetry actually depens on H_{rel}. and its eigenstate is given by

$$\Phi(r_1 r_2) \cdot \chi(r_1 r_2)$$

1. Assume $s = 0$, $\Rightarrow$ boson $\Rightarrow \phi(r_1 r_2)$ symmetric now if $\Psi_{n\ell m}(r, \theta, \phi)$ and the exchange is equivalent to $r \to -r$ which is the same as $\Psi_{n\ell m}(r \to -r, \theta, \phi) = (-1)^\ell\, \Psi_{n\ell m}(r, \theta, \phi)$

$\Rightarrow$ to stay symmetric $\Psi_{n\ell m}$ only can have $\ell =$ even!

2 Now let's say we have 2-electrons $s = 1/2$

As we discussed above we will have

$$\chi(s_1, s_2) = \begin{cases} \chi_s & \text{triplet } \uparrow\uparrow \\ \chi_a & \text{singlet } \downarrow\uparrow \end{cases}$$

Since the total ψ must be antisymm.

we can say that $\psi_{nem}(r, \theta, \phi) = \begin{cases} \text{antisymm for } \chi_s \\ \text{symm. for } \chi_a \end{cases}$

so $\begin{cases} \ell = \text{even} & \text{for } s=0 \\ \ell = \text{odd} & \text{for } s=1 \end{cases}$

overall $\boxed{\ell + s = \text{even}}$

Now if ϕ_{α_1} and ϕ_{α_2} are the spatial wave functions

Singlet $\uparrow\downarrow$: $\phi_s(12) = \frac{1}{\sqrt{2}}\left[\phi_{\alpha_1}(1)\phi_{\alpha_2}(2) + \phi_{\alpha_2}(1)\phi_{\alpha_1}(2)\right]$

triplet $\uparrow\uparrow$: $\phi_a(12) = \frac{1}{\sqrt{2}}\left[\phi_{\alpha_1}(1)\phi_{\alpha_2}(2) - \phi_{\alpha_2}(1)\phi_{\alpha_1}(2)\right]$

Assume now the electrons come very close to each other

$$\left.\begin{array}{l} \phi_{\alpha_1}(1) \approx \phi_{\alpha_1}(2) \\ \phi_{\alpha_2}(1) \approx \phi_{\alpha_2}(2) \end{array}\right\} \Rightarrow$$

$\phi_a(12) \to 0$

This means that probability of 2 electrons (triplet ones) $\uparrow$ $\uparrow$ to come close is very small. Or it may look like they repel each other. This effect is not b/c of their charge but rather from the symmetry consideration of having the overall antisymmetric wave function for fermions.

THIS WHAT WE CALL THE EXCHANGE HOLE!

What about bosons?

in this case $\Phi_s \cong \sqrt{2}\, \phi_{\alpha_1}(1)\, \phi_{\alpha_2}(2)$

which is times 2 over the average value.

Hence 2 non-interacting bosons love to
(weakly)

come together at the same space point if
their eigenstate is symmetric.

So if $(\uparrow\downarrow)$ and $S=0$ they act like
χ_a

they attract each other.

This is the idea behind exchange "force".

It look like the fact of repulsion or attraction depends on what spin state our many-body system is.
$\widehat{\text{total}}$

This kind of interaction is known as exchange interaction and VERY important in condensed matter and especially in strongly correlated electronic matter.

This is purely quantum phenomena and is due to the fact we cannot label the particles in QM.

Read 18.7 on EXCITED STATE OF HE ATOM.

END OF PART I
IDENTICAL PARTICLES