QUANTUM MECHANICS P502 - Spring 2020

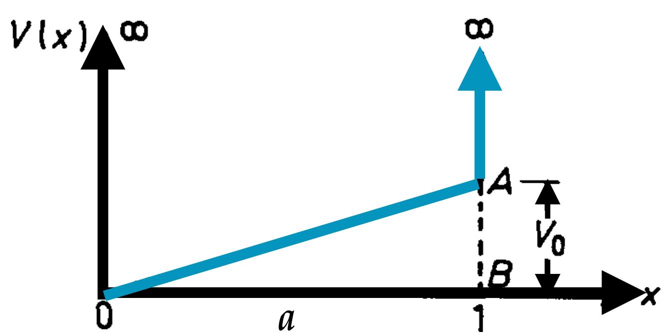
FINAL EXAM PROBLEMS - Due May 13, Wed, 10 am.

Please send your solution via email to *jak.chakhalian at rutgers.edu*
In the SUBJECT line type: P502 final exam and your Rutgers ID.
A single file should be in pdf format, easily readable, with your name on each page and each page numbered sequentially.

P1. N noninteracting bosons are in an infinite potential well defined by $V(x) = 0$ for $0 < x < a$; $V(x) = \infty$ for $x < 0$ and for $x > a$. Find the ground state energy of the system. What would be the ground state energy if the particles are fermions.

P2. Show that Dirac's Hamiltonian for a free particle commutes with the operator $\sigma \cdot p$, where p is the momentum operator and σ is the Pauli spin operator in the space of four component spinors.

P3. Employing first order perturbation theory, calculate the energy of the first three states for an infinite square well of width a, whose portion AB has been sliced off. (Note: The line OA is a straight line).



P4. Consider an electron in a one-dimensional box of length 1 Angstrom.

(a) Find the first 4 wave functions. Normalize the wave functions and sketch them.

(b) Compute the corresponding 4 energy levels and sketch an energy level diagram.

(c) At t = 0, the particle is known to be in the state for which n=1. At t = 0, a rectangular potential well $V_o = -10^4$ eV, centered at a/2 and of width 10^{-12} cm, is suddenly introduced into the well and kept there for 5×10^{-18} secs, at which time it is removed.

After removal of the perturbation, what is the probability that the system will be found in each of the states n=2, n=3, and n=4? (By the way, the height and width of the potential well is characteristic of a neutron interacting with an electron).

P5. The Hamiltonian for an isotropic harmonic oscillator in two dimensions is

$$H = \omega(n_1 + n_2 + 1) \ ,$$

where $n_i = a_i^+ a_i$, with $[a_i, a_j^+] = \delta_{ij}$ and $[a_i, a_j] = 0$.

(a) Work out the commutation relations of the set of operators $\{H, J_1, J_2, J_3\}$ where

$$J_1 = \frac{1}{2}(a_2^+ a_1 + a_1^+ a_2), \quad J_2 = \frac{i}{2}(a_2^+ a_1 - a_1^+ a_2),$$

$$J_3 = \frac{1}{2}(a_1^+ a_1 - a_2^+ a_2).$$

(b) Show that $\mathbf{J}^2 \equiv J_1^2 + J_2^2 + J_3^2$ and J_3 form a complete commuting set and write down their orthonormalized eigenvectors and eigenvalues .

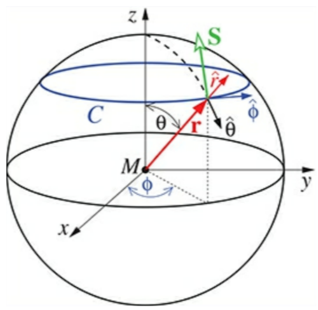
(c) Discuss the degeneracy of the spectrum and its splitting due to a small perturbation $\mathbf{V} \cdot \mathbf{J}$ where $\mathbf{V}$ is a constant three-component vector.

P6. Calculate in the Born approximation to the differential and total cross sections for scattering a particle of mass m off the 3-dimensional delta-function potential $V(\vec{r}) = g\delta^3(\vec{r})$.

SEE NEXT PAGE FOR PROBLEM **P7.**

P7. BERRY'S PHASE FOR SPIN S=1 IN A MAGNETIC MONOPOLE FIELD

A useful model for illustrating the ideas of Berry curvature and Berry's phase consists of a spin interacting with a magnetic monopole (referred to as the 'Dirac monopole') field. Consider a spin $\vec{S} = 1$ particle located at $\vec{r}$. The Dirac monopole of magnitude M situated at the origin produces a magnetic field at the position $\vec{r}$ given by: $\vec{B} = \frac{M}{r^3}\vec{r} = \frac{M}{r^2}\hat{r}$, with $\hat{r}$ the unit vector in the radial direction and $|\vec{r}| = r$ the distance from the origin which we will take to be fixed. See Figure below.



The interaction hamiltonian is: $H = \vec{B} \cdot \vec{S}$.

1. Write components of the hamiltonian H in spherical coordinates.
2. Find eigenstates and eigenvalues (remember this is S=1 problem!).
3. Check that those eigenstates are orthogonal and normalized.
4. What are the eigenstates at the north $\theta = 0$ and π - the south pole?
5. We can now imagine that the spin is moved around a horizontal (constant-latitude) circle, which means θ is fixed and ϕ is between $[0,\, 2\pi]$. What is Berry's phase for this change in position calculated as a contour integral?
6. Calculate the Berry curvature $\Omega_{\theta,\phi}(\theta, \phi)$.
7. Calculate the Berry phase as an surface integral over the entire sphere and set $\theta = \pi$.

Good luck !