

The Haldane Model

INVERSION CENTERS

Forward: t_2
Backwards: t_2^*

① → ②
 $H_{12} \propto t_2 C_2^\dagger C_1 + t_2^* C_1^\dagger C_2$
 complex!
 Term that breaks time reversal symmetry

'SPINLESS GRAPHENE'
 THIS MODEL IS GAPLESS WITH TWO 'DIRAC POINTS'

Left Dirac Point
Right Dirac Point

INVERSION SYMMETRY $A=B$
 Time Reversal symmetry t_2 is 'Real'

BOTH symmetries ARE NEEDED TO 'PROTECT' THE DIRAC POINTS.

2) Break Inversion Symmetry $A \neq B$

- GAP OPENS (Boron Nitride BN semiconductor)
- Just get a "BORING" CONVENTIONAL SEMI CONDUCTOR

Density of states

\downarrow GAP

Why the Haldane Model?

- First crystal model describing topological behavior.

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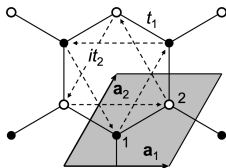
- First crystal model describing topological behavior.
- Simple and intuitive tight binding model.

Why the Haldane Model?

- First crystal model describing topological behavior.
- Simple and intuitive tight binding model.
- Helped me understand what topological materials are all about.

The Model

- Spinless model on the Honeycomb lattice (2 sites)

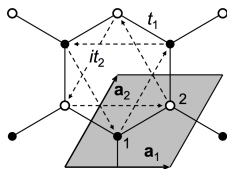


- On-site energy

$$H = \Delta \sum_i (-)^{\tau_i} c_i^\dagger c_i + t_1 \sum_{\langle ij \rangle} (c_i^\dagger c_j + h.c.) + t_2 \sum_{\langle\langle ij \rangle\rangle} (ic_i^\dagger c_j + h.c.)$$

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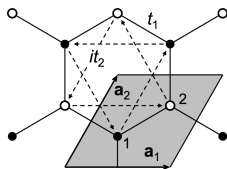
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- Real first neighbor hoppings
- Imaginary second neighbor hoppings

Defining non-trivial states

- The atomic limit: Strength of hoppings and hybridization is zero i.e flat bands

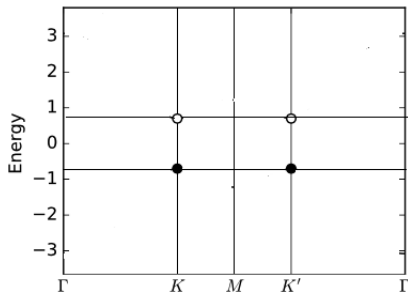


Figure: $\Delta = 0.7, t_1 = t_2 = 0$

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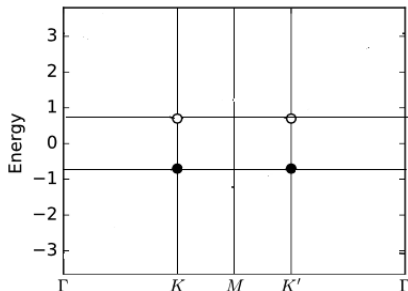
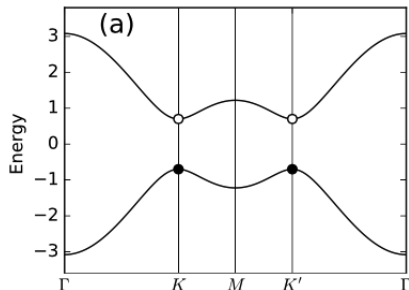


Figure: $\Delta = 0.7, t_1 = t_2 = 0$

- Definition: A state is topological when it is impossible to turn off the interactions (atomic limit) without closing the gap.

Trivial vs Topological

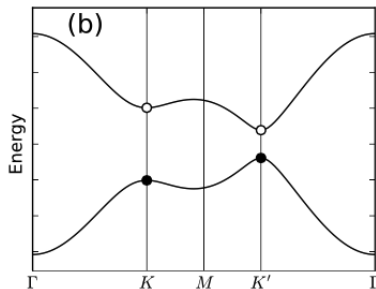
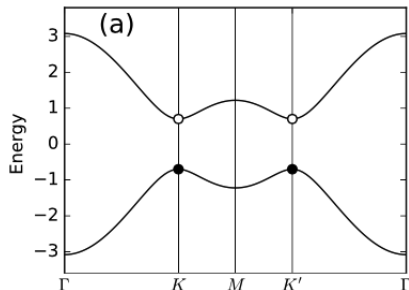
- $\Delta = 0.7$, $t_1 = -1.0$, $t_2 = 0$ and $t_2 = -0.06$



¹Vanderbilt. Berry Phases in Electronic Structure

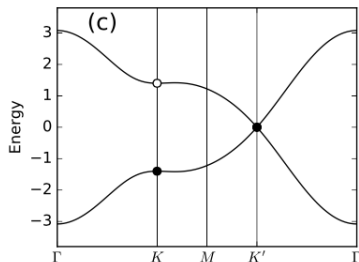
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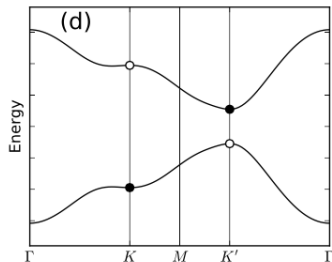
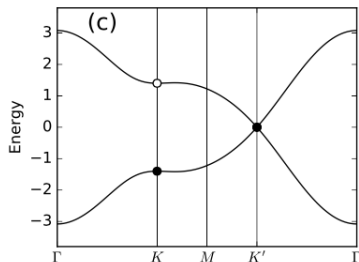
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What is topological about the topological state?

- Hybrid Wannier Representation:

$$|h_{nk_1l_2}\rangle = \frac{1}{2\pi} \int_0^{2\pi} e^{-ik_2l_2} |\psi_{nk_1k_2}\rangle dk_2$$

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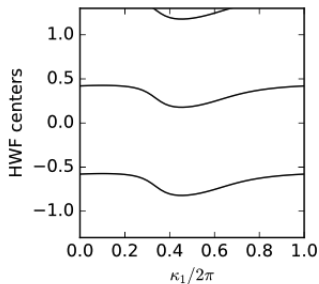
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- Well defined expectation value of the y operator.

Trivial vs Topological

1

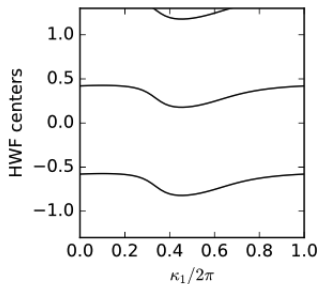


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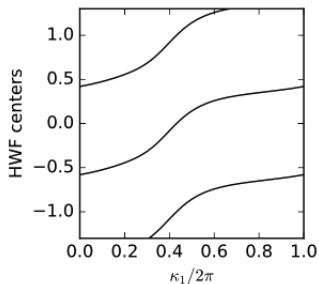
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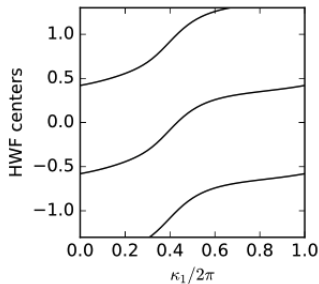


Topological

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Quantized Hall Conductivity

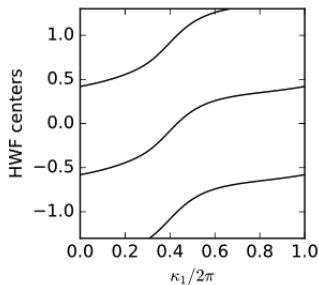
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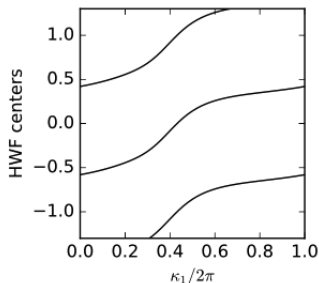


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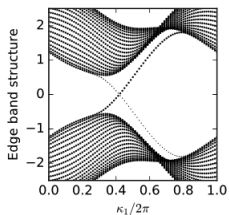
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- C = Chern number = number of unit cells after adiabatic cycle

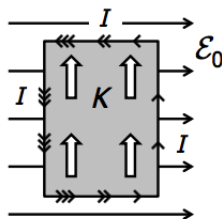


Bulk-Boundary Correspondence

- Charge moving from the bottom to the top surface.

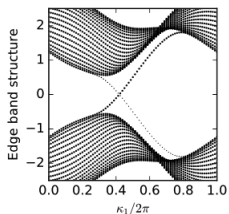


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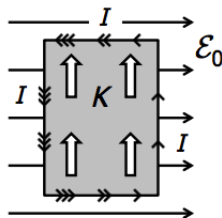


Bulk-Boundary Correspondence

- Charge moving from the bottom to the top surface.
- The only way is if the boundary is conducting.



3



Summary

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- Global properties are quantized.
- **Weak perturbation will not affect them.**

