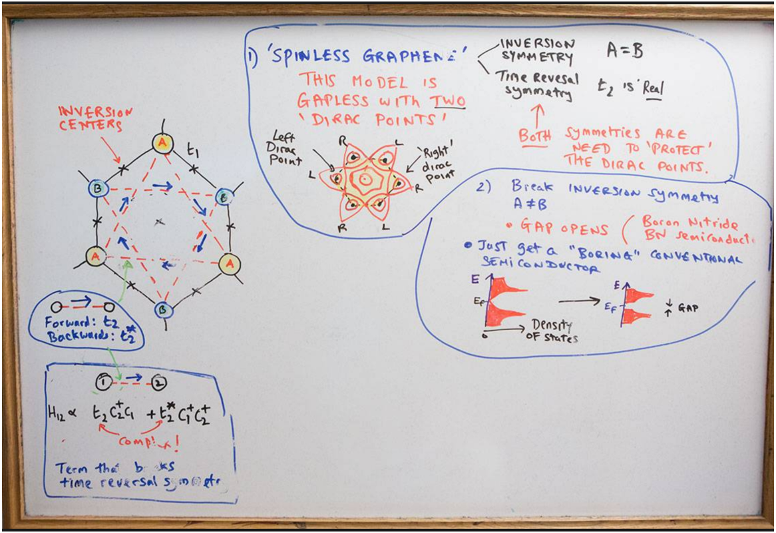
# The Haldane Model



# Why the Haldane Model?

- First crystal model describing topological behavior.

# Why the Haldane Model?

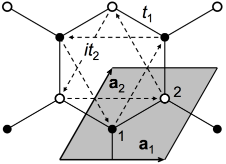
- First crystal model describing topological behavior.
- Simple and intuitive tight binding model.

# Why the Haldane Model?

- First crystal model describing topological behavior.
- Simple and intuitive tight binding model.
- Helped me understand what topological materials are all about.

# The Model

- Spinless model on the Honeycomb lattice (2 sites)

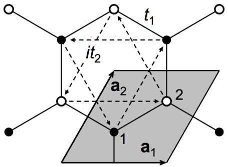



- On-site energy

$$H = \Delta \sum_i (-)^{\tau_i} c_i^\dagger c_i + t_1 \sum_{<ij>} \left(c_i^\dagger c_j + h.c\right) + t_2 \sum_{<<ij>>} \left(i c_i^\dagger c_j + h.c\right.$$

# The Model

- Spinless model on the Honeycomb lattice (2 sites)



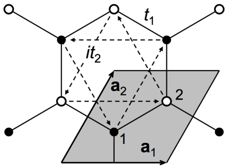

- On-site energy

$$H = \Delta \sum_i (-)^{\tau_i} c_i^\dagger c_i + t_1 \sum_{<ij>} (c_i^\dagger c_j + h.c) + t_2 \sum_{<<ij>>} (i c_i^\dagger c_j + h.c$$

- Real first neighbor hoppings

# The Model

- Spinless model on the Honeycomb lattice (2 sites)




- On-site energy

$$H = \Delta \sum_i (-)^{\tau_i} c_i^\dagger c_i + t_1 \sum_{<ij>} (c_i^\dagger c_j + h.c) + t_2 \sum_{<<ij>>} (i c_i^\dagger c_j + h.c$$

- Real first neighbor hoppings
- Imaginary second neighbor hoppings

- The atomic limit: Strength of hoppings and hybridization is zero i.e flat bands

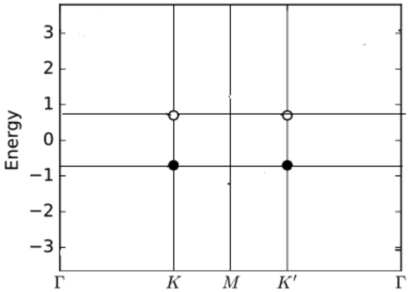



Figure: $\Delta = 0.7, t_1 = t_2 = 0$

# Defining non-trivial states

- The atomic limit: Strength of hoppings and hybridization is zero i.e flat bands

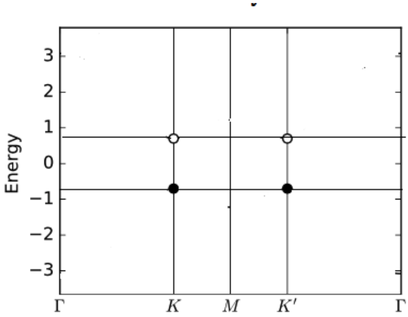
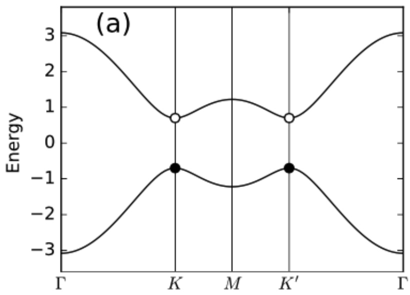



Figure: $\Delta = 0.7, t_1 = t_2 = 0$

- Definition: A state is topological when it is impossible to turn off the interactions (atomic limit) without closing the gap.

# Trivial vs Topological

- $\Delta = 0.7$, $t_1 = -1.0$, $t_2 = 0$ and $t_2 = -0.06$

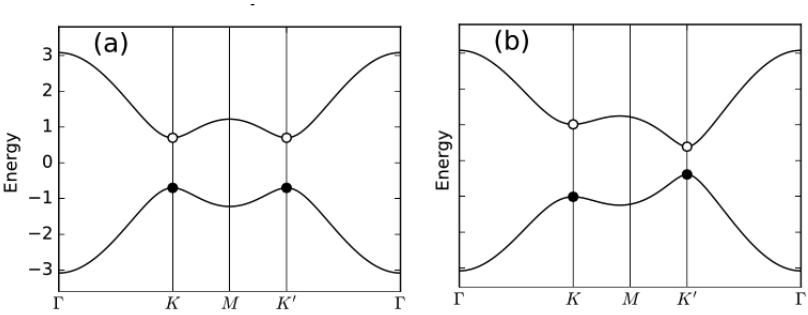



1

[1] Vanderbilt. Berry Phases in Electronic Structure

# Trivial vs Topological

- $\Delta = 0.7$, $t_1 = -1.0$, $t_2 = 0$ and $t_2 = -0.06$



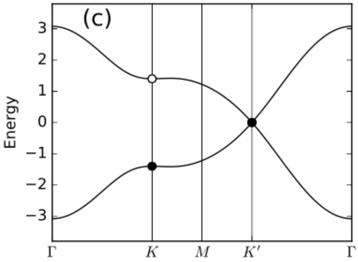
[1]

[1] Vanderbilt. Berry Phases in Electronic Structure

# Trivial vs Topological

- $\Delta = 0.7$, $t_1 = -1.0$, $t_2 = -0.1347$ and $t_2 = -0.24$

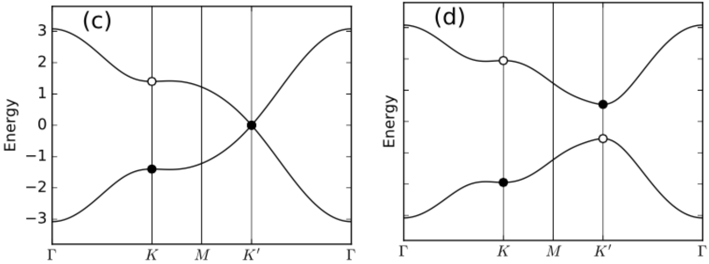
[2]




[2] Vanderbilt. Berry Phases in Electronic Structure

# Trivial vs Topological

- $\Delta = 0.7$, $t_1 = -1.0$, $t_2 = -0.1347$ and $t_2 = -0.24$




[2]

[2] Vanderbilt. Berry Phases in Electronic Structure

# What is topological about the topological state?

- Hybrid Wannier Representation:
  $$|h_{nk_1l_2}\rangle = \frac{1}{2\pi}\int_0^{2\pi} e^{-ik_2l_2}\,|\psi_{nk_1k_2}\rangle\,dk_2$$

# What is topological about the topological state?

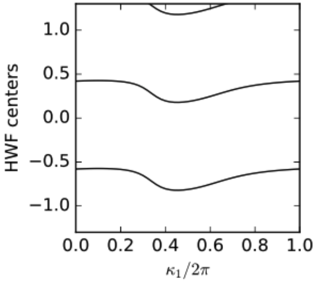
- Hybrid Wannier Representation:
  $$|h_{nk_1l_2}\rangle = \frac{1}{2\pi}\int_0^{2\pi} e^{-ik_2l_2}\,|\psi_{nk_1k_2}\rangle\,dk_2$$
- State $|h_{nk_1l_2}\rangle$ is localized at the $l_2/a$ unit cell in the y direction and extended in x.

# What is topological about the topological state?

- Hybrid Wannier Representation:
  $|h_{nk_1l_2}\rangle = \frac{1}{2\pi}\int_0^{2\pi} e^{-ik_2l_2}\,|\psi_{nk_1k_2}\rangle\, dk_2$
- State $|h_{nk_1l_2}\rangle$ is localized at the $l_2/a$ unit cell in the y direction and extended in x.
- Well defined expectation value of the y operator.

# Trivial vs Topological

1

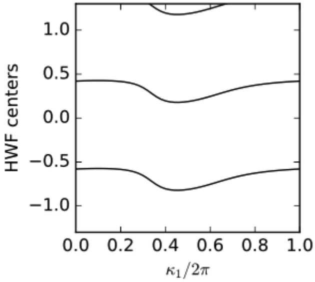



Trivial

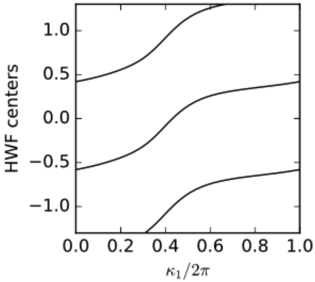
[1] Vanderbilt. Berry Phases in Electronic Structure

# Trivial vs Topological

[1]




Trivial

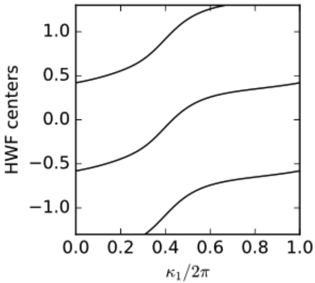



Topological

[1] Vanderbilt. Berry Phases in Electronic Structure

# Quantized Hall Conductivity

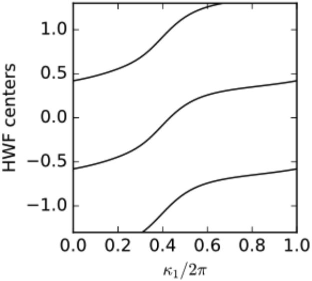
- The semiclassical Boltzmann theory tells as: $\dot{\boldsymbol{k}} = -\frac{e}{\hbar}\boldsymbol{\mathcal{E}}$



# Quantized Hall Conductivity

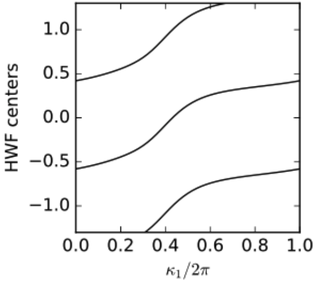
- The semiclassical Boltzmann theory tells as: $\dot{\boldsymbol{k}} = -\frac{e}{\hbar}\boldsymbol{\mathcal{E}}$
- An electric field in the x direction will move states in the y direction

$\sigma_{xy} = C\frac{e^2}{h}$



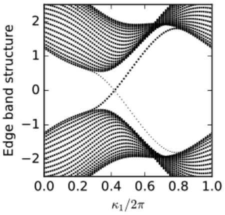
# Quantized Hall Conductivity

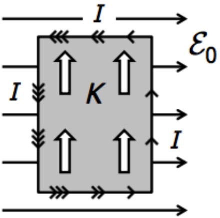
- The semiclassical Boltzmann theory tells as: $\dot{\boldsymbol{k}} = -\frac{e}{\hbar}\boldsymbol{\mathcal{E}}$
- An electric field in the x direction will move states in the y direction $\sigma_{xy} = C\frac{e^2}{h}$
- C = Chern number = number of unit cells after adiabatic cycle



# Bulk-Boundary Correspondence

- Charge moving from the bottom to the top surface.



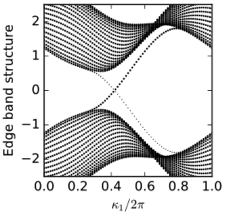



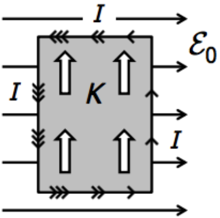

3

[3] Vanderbilt. Berry Phases in Electronic Structure

# Bulk-Boundary Correspondence

- Charge moving from the bottom to the top surface.
- The only way is if the boundary is conducting.




3

[3] Vanderbilt. Berry Phases in Electronic Structure

# Summary

- Topological states cannot be connected to the atomic limit without closing the gap.

# Summary

- Topological states cannot be connected to the atomic limit without closing the gap.
- Global properties are quantized.

# Summary

- Topological states cannot be connected to the atomic limit without closing the gap.
- Global properties are quantized.
- Weak perturbation will not affect them.


Thank You!