

On Goldstone mode

https://en.wikipedia.org/wiki/Spontaneous_symmetry_breaking

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Spontaneously breaking of a continuous symmetry is inevitably accompanied by gapless (meaning that these modes do not cost any energy to excite) Nambu–Goldstone modes associated with slow long-wavelength fluctuations of the order parameter. For example, vibrational modes in a crystal, known as phonons, are associated with slow density fluctuations of the crystal’s atoms. The associated Goldstone mode for magnets are oscillating waves of spin known as spin-waves. For symmetry-breaking states, whose order parameter is not a conserved quantity, Nambu–Goldstone modes are typically mass- less and propagate at a constant velocity.

An important theorem, due to Mermin and Wagner, states that, at finite temperature, thermally activated fluctuations of Nambu–Goldstone modes destroy the long- range order, and prevent spontaneous symmetry breaking in one- and two-dimensional systems.

Similarly, quantum fluctuations of the order parameter prevent most types of continuous symmetry breaking in one-dimensional systems even at zero temperature (an important exception is ferromagnets, whose order parameter, magnetization, is an exactly conserved quantity and does not have any quantum fluctuations).

A more precise statement of this physics is known as Goldstone’s Theorem, which we para- phrase as follows:

If the Hamiltonian (or free energy or Lagrangian) is finite ranged, and if a continuous symmetry is spontaneously broken, then there exist gapless (or “massless”) excitations associated with the local reorientation of the broken symmetry.

Comments:

- The massless excitations are known as (Nambu)-Goldstone bosons.
- If the symmetry is explicitly broken weakly (say, for example, a small external magnetic field is added to the Heisenberg model thus picking out one particular

direction) then the Goldstone bosons become finite energy (“gapped” or “massive”).

- In theories with gauge symmetry the situation is different. Goldstone bosons can get “eaten” by a gauge boson to develop a mass.