Lecture 5

Themes $\Bigg\{$ Fluctuations in the 2^d order parameter,
Critical indices and scaling,
Quantum phase transitions

Close to T_c fluctuations become really important and this leads to modifications in c_p, α, compressibility α etc.
For example instead of a jump in c_p there will be a real singularity, i.e.

$$C(T) \sim \ln(|T-T_c|/T_c) \sim |T-T_c|^{-\alpha}$$

So the real issue is where exactly the fluctuations are really imporant?

answer: $\langle \Delta\eta^2 \rangle \sim \eta^2$

From Ginsburg - Levanyuk theory

$$\tau \sim \frac{|T-T_c|}{T_c} \sim \frac{B^2}{8\pi^2 a^4 T_c^2 \xi_0^6} = \frac{B^2 T_c}{8\pi^2 a G^3}$$

where $\xi_0 \equiv \sqrt{\frac{G}{aT_c}}$ - is known as the correlation length at zero T.

def: The correl. length τ defines a typical scale in the ordered state $T=0$, ~~the equilibrium~~ the order parameter "returns" to the equilibrium value when "destrurbed".

Inside τ fluctuations are so important that they can modify all the thermodynamic quantities.

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To quantify this we introduce the Ginzburg number $Gi \equiv \frac{B^2 T_c}{8\pi^2 a G^3}$

and if $Gi \ll 1$ we can use Landau theory without restrictions, i.e.

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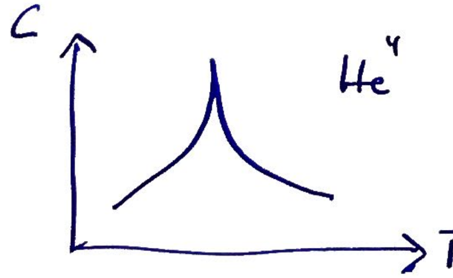
$\tau \sim \frac{|T-T_c|}{c} = Gi \ll 1 \Rightarrow \xi_0$ is large

For example for superconductors

$\xi_0 \sim 10^4 \text{Å}$ – the so-called coherence length is $\gg a \leftarrow$ the lattice parameter.

But for ^4He $\xi_0 \sim a$ and the fluctuations are always super strong.



The Lambda-curve or the λ-anomaly.

An EXPLANATIONS! b/c phase fluctuations are very strong, above T_c the phase doesn't dissipear right away, instead the phase still fluctuates. above T_c. We can call this phenomenon as short-ordering. This in turn means the whole entropy is not released at T_c and some remains above T_c. Also close to T_c fluctuations destroy the expected behaviour for C (when $T \to T_c$)

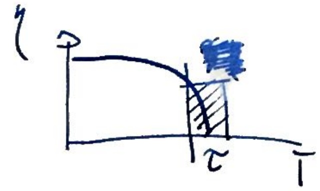
In general: The spatial size of the region where fluctuations are important are defined by the exchange interaction.

e.g. if the interaction ~~of long range~~ of a long range, the $Gi \ll 1$, the fluctuations are small.

A MEANING OF ξ: Consider fluctuations of the order parameter at "0" and "$\vec{r}$" the are correlated

$$\langle \Delta\eta(0) \cdot \Delta\eta(r) \rangle \sim \frac{T}{r} e^{-r/\xi}$$

and $\xi(T) = \sqrt{\dfrac{G}{a|T-T_c|}}$, note $\xi_0 = \xi(T=0)$



within τ b/c of the fluctuations there are regions of phase fluctuations

We can also describe the fluctuations in momentum Space, e.g.

$$\langle \Delta\eta_q \, \Delta\eta_{-q} \rangle = \langle |\eta_q|^2 \rangle \Rightarrow$$

$$T \times \int \frac{e^{-r/\xi}}{r} e^{-ikr} d^3\vec{r} = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \frac{e^{-r/\xi}}{r} e^{-ikr\cos\theta} r^2 \sin\theta \, dr \, d\theta \, d\varphi \underset{\text{in polar coordinates}}{\overset{\text{this is all}}{=}} = 2\pi \int_0^{\pi}\int_0^{\infty} r e^{-r/\xi} e^{-ikr\cos\theta} =$$

$$\sin\theta \, dr \, d\theta, = \underbrace{\text{let } u = \cos\theta \Rightarrow du = -(\sin\theta) d\theta}, =$$

$$= -2\pi \int_1^{-1}\int_0^{\infty} r e^{-r/\xi} e^{-ikru} \, dr \, du = 2\pi \int_{-1}^{+1}\int_0^{\infty} \dots =$$

$= \text{integrate over } u \text{ first} = 2\pi \int_0^{\infty} r e^{-r/\xi} \left[-\frac{1}{ikr} e^{-ikru} \right]_{-1}^{+1} dr$

$$= 2\pi \int_0^{\infty} r e^{-r/\xi} \left[-\frac{e^{-ikr}}{ikr} + \frac{1}{ikr} e^{ikr} \right] dr =$$

$$T * \Big| = \frac{2\pi}{ik}\int_0^\infty e^{-r/\xi}\left[-e^{-ikr} + e^{ikr}\right] dr =$$

$$= \frac{2\pi}{ik}\int_0^\infty \left[-e^{-(ik+\frac{1}{\xi})r} + e^{(ik-\frac{1}{\xi})r}\right] dr =$$

$$= \frac{2\pi}{ik}\left[\frac{e^{-(ik+\frac{1}{\xi})r}}{\frac{1}{\xi}+ik} + \frac{e^{(ik-1/\xi)r}}{ik-\frac{1}{\xi}}\right]\Bigg|_0^\infty =$$

$$= \frac{2\pi}{ik}\left[\frac{e^{-(ik+\frac{1}{\xi})r}}{\frac{1}{\xi}+ik} - \frac{e^{(ik-\frac{1}{\xi})r}}{\frac{1}{\xi}-ik}\right]\Bigg|_0^\infty =$$

$$= \frac{2\pi}{ik}\left[\frac{(\frac{1}{\xi}-ik)e^{-(ik+\frac{1}{\xi})r}}{k^2+\frac{1}{\xi^2}} - \frac{(\frac{1}{\xi}+ik)e^{(ik-\frac{1}{\xi})r}}{k^2+\frac{1}{\xi^2}}\right]_0^\infty$$

$$= \frac{2\pi}{ik}\left[\frac{(\frac{1}{\xi}-ik)e^{-(ik+\frac{1}{\xi})r} - (\frac{1}{\xi}+ik)e^{(ik-\frac{1}{\xi})r}}{k^2+\frac{1}{\xi^2}}\right]_0^\infty =$$

$$= \frac{2\pi}{ik}\left[\frac{1}{\xi}e^{-(ik+\frac{1}{\xi})r} - ik\,e^{-(ik+\frac{1}{\xi})r} - \frac{1}{\xi}e^{(ik-\frac{1}{\xi})r} - ik\cdot e^{(ik-\frac{1}{\xi})r}\right]_0^\infty =$$

$$= \frac{2\pi}{ik}\left[\frac{-\frac{1}{\xi}+ik+\frac{1}{\xi}+ik}{k^2+\frac{1}{\xi^2}}\right] = \frac{2\pi}{ik}\,\frac{2ik}{k^2+1/\xi^2} = \frac{4\pi}{k^2+\frac{1}{\xi^2}}$$

sorry k is the same as q

$$= \frac{4\pi\xi^2}{1+\xi^2k^2}\cdot T$$ So the answer is

$$\boxed{\langle\Delta\eta(q)\rangle^2 \sim \frac{4\pi T\ \xi^2}{1+\xi^2 q^2}} \sim$$

This is a famous Ornstein – Zernike theory of fluctuations.

There are ~~intts~~ interesting relationships between $\langle \eta(q)\rangle^2$ and the response of a system any external perturbation:

In fact we are familiar with some of those $\varepsilon(\bar{q},\omega)$ and $\chi(\bar{q},\omega)$

↑ dielectric constant ↑ magnetic susceptibility.

For static susceptibility $\chi(\bar{q})$

$$\chi(\bar{q}) = \frac{1}{T}\overbrace{\int \frac{d^3\bar{r}}{(2\pi)^3}\, e^{i\bar{q}\cdot\bar{r}}\,\langle \eta(0,t)\,\eta(\bar{r},t)\rangle}^{=\langle \eta(0)\eta(q)\rangle}$$

which is exactly what the Fourier transformation is about.

(see page 4) $\rightarrow \langle \eta(0)\,\eta(q)\rangle = \dfrac{4\pi T\,\xi^2}{1+q^2\xi^2}$

close to T_c

$$\boxed{\chi(\bar{q})\Big|_{T>T_c} = \frac{4\pi\,\xi^2(T)}{1+q^2\xi^2(T)}} = \frac{4\pi G}{a|T-T_c|}\cdot\frac{1}{1+q^2\xi^2}$$

where $\xi(T) = \sqrt{G/a|T-T_c|}$

and for the static measurements in a SQUID magnetometer $\bar{q}=0$, we get

$$\chi(0) = \frac{4\pi G}{a|T-T_c|} \quad \text{or} \quad \boxed{\chi(0) \sim \frac{1}{a|T-T_c|}}$$

for $T>T_c$