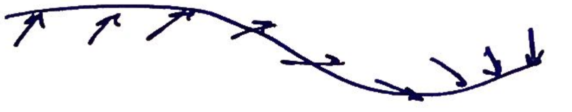
Lecture 4

So far we only considered the case when $\eta(r) = \text{const}$!
But what if $\eta = \eta(r)$ or we would say it's <u>inhomogenious</u>?
In this case the total energy is:

$$\Phi = \int d^3\bar{r}\ (\text{free energy density } \Phi(r))$$

e.g.



~~...~~ spirals, e.g. domains, domain walls, linear defects etc.

NB! The variation of the order parameter in space cost energy!
Slow variation seems the very best approach. i.e. $\frac{du}{dr} \sim \nabla \eta(r)$

The only space invariant which contains ∇ is ∇^2, i.e. $(\nabla\eta)^2$ would go into the free energy term

By direct analogy, the Gibbs free energy can be written as:

$$\boxed{\Phi_F = \int d^3\bar{r} \left\{ A\eta^2(r) + B\eta^4(r) + G(\nabla\eta)^2 \right\}}$$

This is known as Ginsburg-Landau functional or Ginsburg-Landau-Wilson functional.

This form of the functional is super popular and is used in the theory of superconductivity, theory of domain walls etc.

Few words about superconductivity:

if we minimize this fun. w.r.t $\eta(r)$ or ψ we ~~get~~ get something new and not the ordinary self-consistent term $\frac{d\Phi}{d\eta} = 0$

Instead we are going to get the famous Landau-Ginzburg equation:

$$-\frac{1}{2m}\left(-i\hbar\nabla - \frac{2e}{c}\bar{A}\right)^2\psi + A\psi + 2B|\psi|^2\psi = 0$$

here $\bar{A}$ - is the vector potential.

this eq. is VERY similar to the Sch. eqn. but with the extra non-linear term $\sim|\psi|^3$

Now close to T_{sc} where $|\psi|$ is very small we can linearize this equation, and solve exactly like the Sch. eqn.

The very same equation can be applied to the theory of ferromagnets and is known as Landau-Lifshits eqn. to describe domains and domain walls, etc.

Recall for the homogenious solution if G coeff. > 0 then we get a usual 2^d order phase transition.

But what if $G(P,T) < 0$?

in this case there should be a transition from the homogenious solution to the <u>inhomogenious one</u>!

e.g. $\uparrow \uparrow \uparrow \uparrow$ $\longrightarrow$ may go to the spiral state.

Now to find the properties like this we move to the min. w.r.t $\nabla\eta$

Also note if $T < T_c$ $A<0$, $G<0$

so to stalize the system we need to add the next positive term: $E(\nabla^2\eta)^2$ with $E>0$ for $T<T_c$

Now we have:

$$\Phi = \int d\vec{r}^3 \left\{ A\eta^2 + B\eta^4 + G(\nabla\eta)^2 + E(\nabla^2\eta)^2 \right\}$$

To make our life simpler let's move into momentum space e.g. $(\nabla\eta)^2 \rightarrow q^2\eta^2$

$$\left.\begin{aligned} \eta(r) &= \tfrac{1}{2\pi}\int dq\, e^{-iqr}\eta(q) \\ \eta(q) &= \tfrac{1}{2\pi}\int dr\, e^{+iqr}\eta(r) \end{aligned}\right\}$$

e.g $\nabla(\eta(r)) = \frac{1}{2\pi}\, iq \int dq\, e^{-iqr}\eta(q) =$

$= -iq\,\eta(r)$

$$\Phi = \int \dots \underline{G(q^2)\eta^2 + E(q^2)^2\eta^2}$$

$$\frac{d\Phi}{\partial(q^2)} = G\eta^2 + 2Eq^2\eta^2 = 0$$

$$\Rightarrow \boxed{q^2_{min} \equiv Q = -\frac{G}{2E} \quad (G<0)}$$

this means that a new structure with a period $\frac{2\pi}{q} = L$ will be formed.

Notice this L doesn't have to be ~~comme~~ commensurate with the ~~[illegible]~~ sample lattice period $|\bar{a}|$.

The point P^*, T^* where $G(P^*, T^*)$ changes its sign is called the LIFSHITS POINT.

Here is a typical phase diagramm for the system with this kind of transition: