# Electrons in magnetic field

Thursday, November 1, 2018 10:00 PM

Long wave length limit is very general.

Assume the amplitude of the w.f. depends on $\bar{r} = x, y$

$$\psi(r) = \begin{pmatrix} c_1(r) \\ c_2(r) \end{pmatrix}$$ in the long $\lambda$ limit:

$$(v_F\, \bar{p}\cdot\bar{\sigma})\psi(r) = E\psi(r) \Rightarrow v_F \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$

$$\begin{pmatrix} c_1(r) \\ c_2(r) \end{pmatrix} = E \begin{pmatrix} c_1(r) \\ c_2(r) \end{pmatrix}$$ with $p_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$ ...

As above for graphene the solutions are $E_-^+(p)$

The eigenstates are given by $\psi(r) = e^{\frac{i}{\hbar} p\cdot r}\,\psi_p$

Mathematically it looks like we have the Dirac eqn. in 2D

$$\left(c\,\bar{p}\cdot\bar{\sigma} + mc^2\sigma_z\right)\psi(r) = E\,\psi(r)$$

speed of light

rest mass

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

But Dirac eqn. is for $s = 1/2$ fermions with $v = c$

in graphene $v_F \sim 0.003c$

For Dirac particles the components of the spinor refer to amplitudes of spin ↑ and spin ↓, while for graphene it's the amplitude of $\psi$ on carbon 1 & 2.

Next we consider the Dirac eqn. when we apply an external magnetic field:

From quantum mechanics we know that we should not use $\bar{B}$ & $\bar{E}$ and instead use $\varphi$ and $\bar{A}$

In this case to keep the eqn. gage invariant we replace $p \to p + eA$

In we apply the field along $z$ $B = (0,0,B) \Rightarrow B = \nabla \times A$ we use $A = (-yB, 0, 0)$. This is what is knows as the Landay gage.

Thus we end up with:

$$v_F \begin{pmatrix} 0 & p_x - ip_y - eyB \\ p_x + ip_y - eBy & 0 \end{pmatrix} \begin{pmatrix} c_1(r) \\ c_2(r) \end{pmatrix} = E \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

Lets write $c_1(r) = e^{ikx} \chi_1(y)$ $\qquad p_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$ $\quad p_y = \frac{\hbar}{i}\frac{\partial}{\partial y}$

$c_2(r) = e^{ikx} \chi_2(y)$

$$\begin{cases} \overbrace{v_F \left(\hbar k - \hbar \frac{\partial}{\partial y} - eBy\right)}^{\equiv \hat{Q}} \chi_2(y) = E \chi_1(y) \\ \underbrace{v_F \left(\hbar k + \hbar \frac{\partial}{\partial y} - eBy\right)}_{\equiv \hat{R}} \chi_1(y) = E \chi_2(y) \end{cases}$$

e.g. $\times E$ $\quad \begin{cases} \hat{Q} \chi_2 = E \chi_1 \\ \hat{R} \chi_1 = E \chi_2 \end{cases} \Rightarrow \hat{R}\hat{Q}\chi_2 = R E \chi_1 = E \hat{R} \chi_1 = E^2 \chi_2$

So we end up:

$$\hat{R}\hat{Q} = v_F\left(\hbar k + \hbar \frac{\partial}{\partial y} - eBy\right) v_F \left(\hbar k - \hbar \frac{\partial}{\partial y} - eBy\right)$$

$$= v_F^2 \left(-\hbar^2 \frac{\partial^2}{\partial y^2} + e^2B^2\left(y - \frac{\hbar k}{eB}\right)^2 - \hbar e B\right) \Rightarrow$$

$$v_F^2 \left(-\hbar^2 \frac{\partial}{\partial y^2} + e^2 B^2 \left(y - \frac{\hbar k}{eB}\right)^2 - \hbar e B\right) \chi_2(y) = E^2 \chi_2(y)$$

Looks difficult BUT:

lets introduce few new variables:

lets introduce few new variables:

$$v_F^2 \equiv \frac{1}{2m} \qquad \omega = 2eBv_F^2 \qquad y' \equiv y - \frac{\hbar k}{eB}$$

$$\boxed{\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2} + \frac{1}{2}m\omega y'^2 - \frac{1}{2}\hbar\omega\right)U(y') = E^2 U(y')}$$

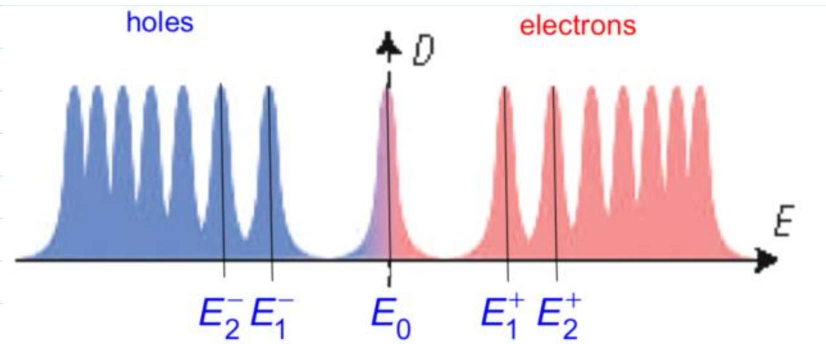
This is just a quantum harmonic oscillator!

with the solution:

$$E^2 = (n + 1/2)\hbar\omega - \frac{1}{2}\hbar\omega = n\hbar\omega \Rightarrow$$

$$\boxed{E_n^{\pm} = \pm\sqrt{n\hbar\omega} = \pm\sqrt{n\hbar eB}\cdot v_F \qquad n = 0, 1, 2 \text{ etc.}}$$

$$E_n^{\pm} \sim \sqrt{n}\cdot\sqrt{B}$$




- Note we have no $\frac{\hbar\omega_0}{2}$ term and $E_0 = 0$
- Also there are $E^+$ (electrons) $E^-$ (holes)
- In undoped graphene $E_F = 0$, this means the lowest Landau level is 1/2 filled
- The eigenstates $\chi_{2,n}(y) = U_n(y - y_k)$ and for given $\chi_2$ we get $\chi_{1,n}(y) = \frac{1}{E}\hat{Q}\chi_{2,n}(y) =$

$$= U_{n-1}(y - y_k)$$

in short $\psi_{0,k}(\vec{r}) = e^{ikx}\begin{pmatrix} 0 \\ v_0(y-y_k) \end{pmatrix}$

$$\psi^{\pm}_{n,k} = \frac{e^{ikx}}{\sqrt{2}}\begin{pmatrix} \pm v_{n-1}(y-y_k) \\ v_n(y-y_k) \end{pmatrix} \qquad n \geq 1$$

We will study the electrons in magnetic field when we talk about the Integer Quantum Hall Effect.