Fermi electrons in magnetic field

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by Jak Chakhalian

Lets Submerge our electron into a solid and apply a external field B

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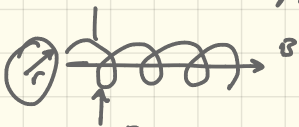
The eq. of motion is given by:

$$\frac{dp}{dt} = -e\, \bar{v} \times \bar{B}$$

classical momentum

velocity

$\Rightarrow$ the particle moves by spiral in FREE SPACE!



$$r_B = \frac{p_\perp}{eB}$$

The condition that we can still use the quasiclassical approximation:

$$\lambda \ll r_B$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi\hbar}{p} \Rightarrow \frac{2\pi\hbar}{p} \ll \frac{p_\perp}{eB}$$

$$\hbar B \ll \frac{p_\perp p}{2\pi} \quad \Big| \times \frac{1}{m_0}$$

So called the cyclotrone frequency $\longrightarrow$ $$\omega_c \equiv \frac{\hbar B}{m_0} \ll \frac{p_\perp p}{2\pi m_0}$$

Now recall, $\bar{p}$ inside the crystall cannot change unless we apply the external forces.

So

$$\frac{dp}{dt} = -e\, v \times B$$ is still ok if we assume p – is a quasi momentum

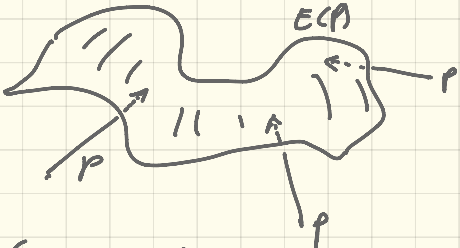
$$\left(v \cdot \frac{dp}{dt}\right) = -e\, v \cdot (v \times B) = 0$$ and

$$\left(B \cdot \frac{dp}{dt}\right) = \dots = 0$$

$\left(v \frac{dp}{dt}\right)$ = conservation o energy

i.e. $v = \frac{dE}{dp}$ and $\left(v \cdot \frac{dp}{dt}\right) = \left(\frac{dE}{dp}\frac{dp}{dt}\right) = \frac{dE}{dt} = 0$

since $F_{Lor.} \perp v$, this means that that the tip of the $\bar{p}$ vector glides on the surface $E(\bar{p}) = const$



From $\left(B \frac{dp}{dt}\right) = 0$ we get

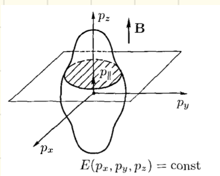
$$\frac{dp}{dt} = \frac{dp_{\parallel}}{dt} + \frac{dp_{\perp \text{ to } B}}{dt} \Rightarrow$$

$$\Rightarrow \left(B \frac{dp_{\parallel}}{dt}\right) = 0 \quad \text{or} \quad \left|\left(\frac{dp}{dt}\right)_{\parallel}\right| |B| = 0 \Rightarrow \left(\frac{dp}{dt}\right)_{\parallel} = 0$$

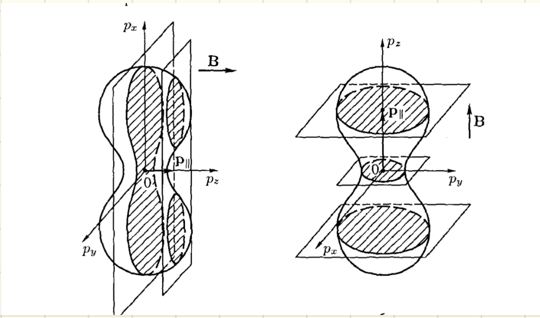
⇒ a projection of the momentum on the direction of B is conserved

$$\begin{cases} P_{\parallel} = const \\ E(P) = const \end{cases}$$

the common solution gives the curve which is the result of a cut of E(p) by a plane which is ⊥ to the magnetic field



Dependin on topology of the F.S. we may end up with a closed or open trajectories.
Here is the example of two types of trajectories:



Trajectory in real space

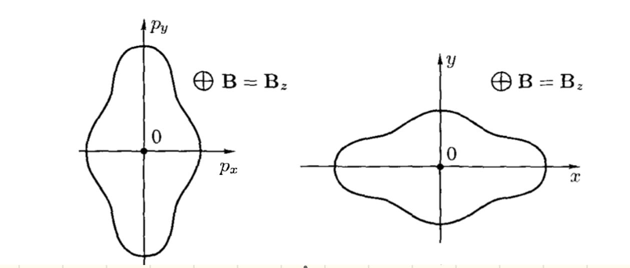
Statement: Trajectory of a quasiparticle in p-space defines its trajectory in r-space. To show this I will project p on the plane $\perp$ to B.

$$\frac{dp_\perp}{dt} = e\, V_\perp \times \bar{B} = e \frac{dr}{dt} \times B \quad , \quad \frac{dr_\perp}{dt} \perp B$$

$|p_\perp| = e|B| \cdot |r_\perp|$ 1) $p_\perp$ scales with $r_\perp$!

2) since $v_\perp = \frac{dr_\perp}{dt}$ in r-space $\perp$ to $\frac{dp_\perp}{dt}$ in p-space ($\Leftarrow$ from $\frac{dp_\perp}{dt} = e \frac{dr_\perp}{dt} \times B$)

This means each element of projection in r-space $\perp$ each element in p-space, i.e. the trajectories are turned 90° wrt each other.



In short to see the trajectory in r-space rotate the plane by 90° and scale it by $\frac{1}{eB}$ times. The direction of motion is the same

Lets estimate a characteristic size of a trajectory in the xtal.

recal $\lambda_B \ll \frac{p_F}{eB}$ $\quad p_F \sim \frac{\hbar}{a}$ $\quad l_B \sim a$

then $B \ll B_a = \frac{\hbar}{e a^2} \sim 10^4 - 10^5\ T$

The largest dc field $\sim 50T$!

A condition for a cyclical motion in the field is $l > r = p_\perp / eB$, or within l at
$\uparrow$ m.f.p

least 1 turn must be completed. Now lets replace $p_F \sim \hbar/a$; $l > \hbar/eaB \Rightarrow$

$$B > \frac{a}{l} \frac{\hbar}{a^2 |e|} \approx \frac{a}{l} B_a \approx \frac{a}{l} \cdot (10^4 - 10^5)\,T$$

since for pure metals $l \sim 10^3 \div 10^5 \cdot a \Rightarrow$ the cyclical trajectory is found already at few Tesla!

Energy Spectrum of quasiparticles in magnetic field

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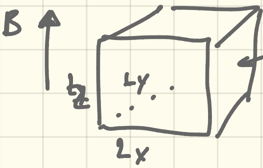
Ideal gas of electrons:



box of free electrons

$$E(\bar{p}) = \frac{|p|^2}{2m_0}$$ let's separate those

$$E = \frac{p_x^2 + p_y^2}{2m_0} + \frac{p_z^2}{2m_0} = E_\perp + E_\parallel$$

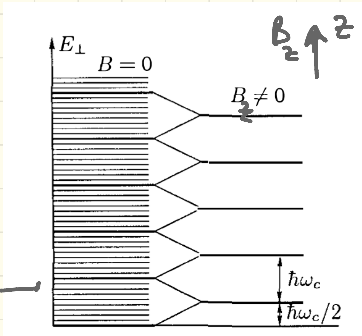
Recall the density of states for 2D is const(E)

$$\nu^{2D}(E) = \frac{m^*}{\pi\hbar^2}$$

- Every energy level is degenerate

for each E we have many p_x and p_y such as $p_x^2 + p_y^2 = 2m_0 E_\perp$

for many n_x and n_y



in the plane $\perp$ to $\bar{B}$ electrons move on the circle of $r_B = \frac{m_0 v_\perp}{Be}$ with $\omega_c = \frac{eB}{m_0}$

↳ the energy is quantized:

$$E = E_\perp + E_\parallel = \hbar\omega_c\left(n + \frac{1}{2}\right) + \frac{p^2}{2m_0}$$

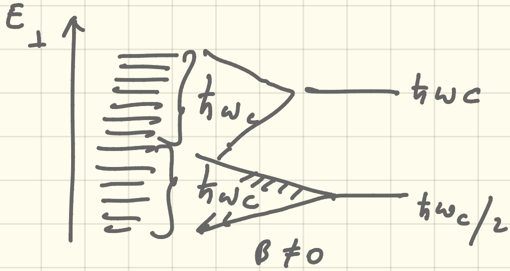
$$n = 0, 1, 2 \ldots$$

For energy $E_\perp$ we have only energies $= \hbar\omega_c(n + 1/2)$ separated by $\hbar\omega_c$

For $\parallel$ we get $E_\parallel = \frac{p_z^2}{2m_0} = \frac{\hbar^2}{2m_0}\left(\frac{2\pi\hbar}{L_x}\right)^2 n_z^2$

huge # of states almost quazicontinious.

See page 5



so already degenerate spectrum of $B=0$ now is VERY degenerate

The # of e^- in the band of size $\hbar\omega_c$

$$N_L = \underbrace{\nu^{2D}(E)}_{\substack{\text{density}\\ \text{of 2D states}}} \cdot \underbrace{\hbar\omega_c}_{\Delta E} = \frac{\overset{= L_x \cdot L_y}{S}\, m_0}{\pi\hbar^2} \cdot \hbar\omega_c = \frac{L_x L_y m_0 \hbar\omega_c}{\pi\hbar^2}$$

N_L defines the degree of degeneracy of $E_\perp$ for $B \neq 0$.

For descrete values of $E_n^\perp$ in the quaziclassical approximation corresponds a specific trajectory; which depends on the quantum # n. Then our condition

$\lambda_B \ll r_{B_n}$ is equal $\hbar\omega_c \ll E_F$

To find the radius r_{B_n} lets compare

$$E_\perp^{classical} = \frac{m_0 \omega_c^2 r_{B_n}^2}{2} \quad \text{and} \quad E_\perp^{quantum} = \hbar\omega_c(n+\tfrac{1}{2})$$

From this we get

$$\boxed{r_{B_n} = \sqrt{\frac{2\hbar}{m_0\omega_c}\left(n+\frac{1}{2}\right)} = \sqrt{\frac{2\hbar}{eB}(n+1/2)}}$$

- So for the electron to go from the orbit n to $n+1$ needs to get a kick $\hbar\omega_c$
- For the same n e^- get the same r_{B_n}. But n_z and p_z can be different

LETS INCLUDE SPIN

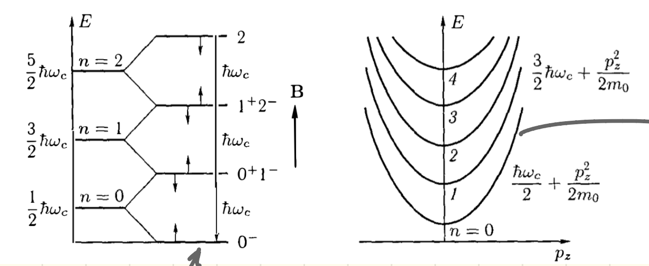
For electron with mag. moment $\mu_B = e\hbar/2m_0c$ its energy in $B = -\bar{\mu}\bar{B}$; with the spin we split a Landau level into 2 sub levels dependin if $\mu \uparrow\uparrow B$ or $\mu \uparrow\downarrow B$

$$E(n,s,k_z) = \hbar\omega_c(n+1/2) + \underset{\substack{\swarrow \\ s=\pm1}}{s}\mu_B B + \frac{\hbar^2 k_z^2}{2m_0}$$

$s = +1 \Rightarrow$ "+" state
-1 .... "−" ... $\}$ the lowest level 0^-

Note: Spin removes the Landau degeneracy for the same n we have $n, s=+1$ $n; s=-1$

(see page 8)

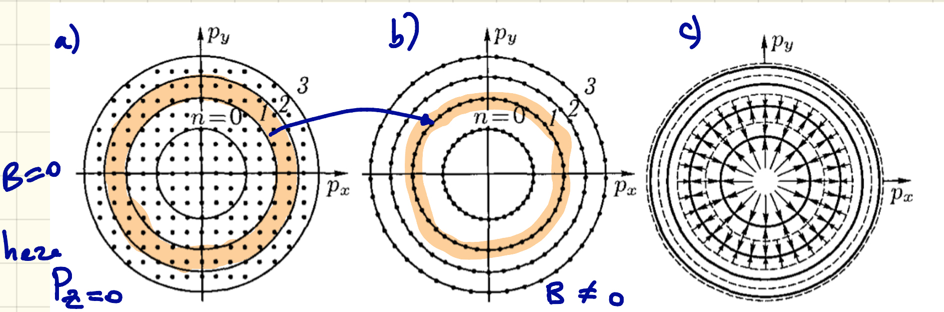


each state on the parabola is strongly degenerate

not spin degeneracy is only absent for 0^-.

Since E continiously depends only on p_z it looks like we have a quasi-1D system!

DISTRIBUTION OF ELECTRONS in p-space



Assume for have a single zone metal with a spherical Fermi surface.
if $B=0$ all the states are inside the sphere and occupy $(2\pi\hbar)^3$. So we mark the py points separated by $2\pi\hbar$. The maximum circle is $p_F = \sqrt{2m^* E_F}$. For any other x-section the states fill up the circle of $\sqrt{p_F^2 - p_z^2}$; as $p_z \to p_F$ the radius goes $\to 0$.

The uniform distribution of states with $p_x, p_y\ p_z$ corresponds to $E = E(p_x\ p_y\ p_z)$ where $0 < |p| < p_F$

Now we turn on B: in the plane $\hbar\omega_c(n + 1/2)$ for $p_z = \text{const}$, to find the radius p_n we write down

$$E_\perp^{classic} = \frac{p_x^2 + p_y^2}{2m^*} = \frac{p_n^2}{2m^*}$$

$$= E_\perp^{quantum} = \frac{1}{2}\hbar\omega_c(n + 1/2)$$

$$\Rightarrow \quad p_n = \sqrt{2m\hbar\omega_c(n + 1/2)}$$ (see fig b in page 8)

In other words: all states which we had confined inside the orbits with a radiu p_n $n = 0, 1, 2, \ldots$, now collapse on the circles see fig. a vs. b in page 8

Note the area in a) $\pi(p_{n+1}^2 - p_n^2) =$

$$= 2\pi \underbrace{m^*\hbar\omega_c}_{r_{Bn}}$$

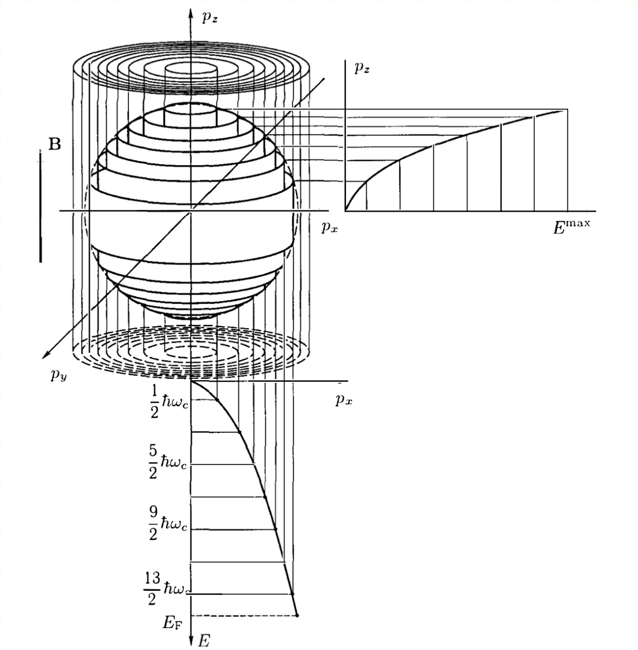
Except for 0^- state:

$$\pi p_0^2 = \pi m^* \hbar\omega_c$$

So for each allowed orbit we have the same # of e^- $N_L = \dfrac{m^* L_x L_y \hbar\omega_c}{\pi\hbar^2} \Rightarrow$

degeneracy of those p_n orbits is the same as the descrete Landau levels

Note since p_n is independent of p_z all orbits are of the same radius then we deal with the Landau cylinders



- number of states filled up by e^- on each cylinder depends on its length within p_F
- with increasing p_n length ↓
- # of cylinders ↓ with increasing ↑

To be cont'd