HW #1

1. Bishop 3.6

2. Bishop 3.8

3. [Bayesian curve fitting and model selection]

(a) Consider Bessel functions of the 1st kind:

$$J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+\alpha+1)} \left(\frac{x}{2}\right)^{2m+\alpha}.$$

Use $t = J_0(x) + \epsilon$ to generate $N = 200$ datapoints in the $(0, 20]$ range, with $x = 0.1, 0.2, \ldots, 20.0$. ϵ is a random variable sampled from $\mathcal{N}(\mu = 0, \underbrace{0.0025}_{\sigma^2 \Rightarrow \sigma = 0.05})$

Plot your dataset superimposed on $J_0(x)$ (i.e., the true function)

(b) Model the dataset from (a) using a polynomial fit:

$$y(x, \vec{w}) = \sum_{j=0}^{M} w_j x^j .$$

Carry out Bayesian regression and plot the mean of the predictive distribution for $M = 1, 2, 3, 4, 5, 6$ superimposed on $J_0(x)$ (plot in two panels if necessary)

For each value of M, find $(\hat{\alpha}, \hat{\beta})$ under the evidence approximation. Compare the predicted values of $\hat{\beta}$ for each M with $\sigma^{-2} = 400$.

(c) Carry out Bayesian model comparison by plotting the log of the model evidence vs. M, as was done in Bishop Fig. 3.14. Out of the six models, which one would you choose and why? Comment on the main features of the plot.